
Contents

| | |
|----------------------|-----|
| Series preface | vii |
| Preface | ix |

Part I Modeling of mechanical systems

| | | |
|----------|--|----|
| 1 | Introductory examples and problems | 3 |
| 1.1 | Rigid body systems | 4 |
| 1.2 | Manipulators and multi-body systems | 6 |
| 1.3 | Constrained mechanical systems | 8 |
| 1.4 | Bibliographical notes | 10 |
| 2 | Linear and multilinear algebra | 15 |
| 2.1 | Basic concepts and notation | 15 |
| 2.1.1 | Sets and set notation | 16 |
| 2.1.2 | Number systems and their properties | 16 |
| 2.1.3 | Maps | 17 |
| 2.1.4 | Relations | 19 |
| 2.1.5 | Sequences and permutations | 19 |
| 2.1.6 | Zorn's Lemma | 20 |
| 2.2 | Vector spaces | 21 |
| 2.2.1 | Basic definitions and concepts | 21 |
| 2.2.2 | Linear maps | 24 |
| 2.2.3 | Linear maps and matrices | 26 |
| 2.2.4 | Invariant subspaces, eigenvalues, and eigenvectors | 29 |
| 2.2.5 | Dual spaces | 30 |
| 2.3 | Inner products and bilinear maps | 33 |
| 2.3.1 | Inner products and norms | 33 |
| 2.3.2 | Linear maps on inner product spaces | 35 |
| 2.3.3 | Bilinear maps | 36 |

| | | |
|----------|---|-----------|
| 2.3.4 | Linear maps associated with bilinear maps | 39 |
| 2.4 | Tensors | 40 |
| 2.4.1 | Basic definitions | 41 |
| 2.4.2 | Representations of tensors in bases | 42 |
| 2.4.3 | Behavior of tensors under linear maps | 43 |
| 2.5 | Convexity | 44 |
| 3 | Differential geometry | 49 |
| 3.1 | The prelude to differential geometry | 50 |
| 3.1.1 | Topology | 51 |
| 3.1.2 | Calculus in \mathbb{R}^n | 56 |
| 3.1.3 | Convergence of sequences of maps | 59 |
| 3.2 | Manifolds, maps, and submanifolds | 60 |
| 3.2.1 | Charts, atlases, and differentiable structures | 60 |
| 3.2.2 | Maps between manifolds | 66 |
| 3.2.3 | Submanifolds | 68 |
| 3.3 | Tangent bundles and more about maps | 70 |
| 3.3.1 | The tangent bundle | 70 |
| 3.3.2 | More about maps | 73 |
| 3.4 | Vector bundles | 77 |
| 3.4.1 | Vector bundles | 78 |
| 3.4.2 | Tensor bundles | 83 |
| 3.5 | Vector fields | 84 |
| 3.5.1 | Vector fields as differential operators | 85 |
| 3.5.2 | Vector fields and ordinary differential equations | 89 |
| 3.5.3 | Lifts of vector fields to the tangent bundle | 94 |
| 3.6 | Tensor fields | 95 |
| 3.6.1 | Covector fields | 96 |
| 3.6.2 | General tensor fields | 98 |
| 3.7 | Distributions and codistributions | 104 |
| 3.7.1 | Definitions and basic properties | 104 |
| 3.7.2 | Integrable distributions | 105 |
| 3.7.3 | The Orbit Theorem for distributions | 108 |
| 3.7.4 | Codistributions | 110 |
| 3.8 | Affine differential geometry | 111 |
| 3.8.1 | Definitions and general concepts | 112 |
| 3.8.2 | The Levi-Civita affine connection | 114 |
| 3.8.3 | Coordinate formulae | 116 |
| 3.8.4 | The symmetric product | 118 |
| 3.9 | Advanced topics in differential geometry | 119 |
| 3.9.1 | The differentiable structure of an immersed submanifold | 120 |
| 3.9.2 | Comments on smoothness, in particular analyticity | 121 |
| 3.9.3 | Properties of generalized subbundles | 123 |
| 3.9.4 | An alternative notion of distribution | 125 |
| 3.9.5 | Fiber bundles | 130 |

| | | |
|----------|--|------------|
| 3.9.6 | Additional topics in affine differential geometry | 131 |
| 4 | Simple mechanical control systems | 141 |
| 4.1 | The configuration manifold | 143 |
| 4.1.1 | Interconnected mechanical systems | 143 |
| 4.1.2 | Finding the configuration manifold | 146 |
| 4.1.3 | Choosing coordinates | 152 |
| 4.1.4 | The forward kinematic map | 155 |
| 4.1.5 | The tangent bundle of the configuration manifold | 157 |
| 4.2 | The kinetic energy metric | 162 |
| 4.2.1 | Rigid bodies | 162 |
| 4.2.2 | The kinetic energy of a single rigid body | 166 |
| 4.2.3 | From kinetic energy to a Riemannian metric | 168 |
| 4.3 | The Euler–Lagrange equations | 172 |
| 4.3.1 | A problem in the calculus of variations | 173 |
| 4.3.2 | Necessary conditions for minimization—the Euler–Lagrange equations | 174 |
| 4.3.3 | The Euler–Lagrange equations and changes of coordinate | 176 |
| 4.3.4 | The Euler–Lagrange equations on a Riemannian manifold | 178 |
| 4.3.5 | Physical interpretations | 182 |
| 4.4 | Forces | 187 |
| 4.4.1 | From rigid body forces and torques to Lagrangian forces | 188 |
| 4.4.2 | Definitions and examples of forces in Lagrangian mechanics | 189 |
| 4.4.3 | The Lagrange–d’Alembert Principle | 193 |
| 4.4.4 | Potential forces | 195 |
| 4.4.5 | Dissipative forces | 198 |
| 4.5 | Nonholonomic constraints | 198 |
| 4.5.1 | From rigid body constraints to a distribution on Q | 199 |
| 4.5.2 | Definitions and basic properties | 200 |
| 4.5.3 | The Euler–Lagrange equations in the presence of constraints | 204 |
| 4.5.4 | Simple mechanical systems with constraints | 207 |
| 4.5.5 | The constrained connection | 209 |
| 4.5.6 | The Poincaré representation of the equations of motion | 213 |
| 4.5.7 | Special features of holonomic constraints | 215 |
| 4.6 | Simple mechanical control systems and their representations | 218 |
| 4.6.1 | Control-affine systems | 218 |
| 4.6.2 | Classes of simple mechanical control systems | 221 |
| 4.6.3 | Global representations of equations of motion | 224 |
| 4.6.4 | Local representations of equations of motion | 225 |
| 4.6.5 | Linear mechanical control systems | 227 |
| 4.6.6 | Alternative formulations | 229 |

| | | |
|----------|---|-----|
| 5 | Lie groups, systems on groups, and symmetries | 247 |
| 5.1 | Rigid body kinematics | 248 |
| 5.1.1 | Rigid body transformations | 249 |
| 5.1.2 | Infinitesimal rigid body transformations | 252 |
| 5.1.3 | Rigid body transformations as exponentials of twists | 254 |
| 5.1.4 | Coordinate systems on the group of rigid displacements | 255 |
| 5.2 | Lie groups and Lie algebras | 258 |
| 5.2.1 | Groups | 258 |
| 5.2.2 | From one-parameter subgroups to matrix Lie algebras | 261 |
| 5.2.3 | Lie algebras | 263 |
| 5.2.4 | The Lie algebra of a Lie group | 265 |
| 5.2.5 | The Lie algebra of a matrix Lie group | 268 |
| 5.3 | Metrics, connections, and systems on Lie groups | 271 |
| 5.3.1 | Invariant metrics and connections | 271 |
| 5.3.2 | Simple mechanical control systems on Lie groups | 275 |
| 5.3.3 | Planar and three-dimensional rigid bodies as systems on Lie groups | 277 |
| 5.4 | Group actions, isometries, and symmetries | 283 |
| 5.4.1 | Group actions and infinitesimal generators | 283 |
| 5.4.2 | Isometries | 288 |
| 5.4.3 | Symmetries and conservation laws | 290 |
| 5.4.4 | Examples of mechanical systems with symmetries | 293 |
| 5.5 | Principal bundles and reduction | 296 |
| 5.5.1 | Principal fiber bundles | 297 |
| 5.5.2 | Reduction by an infinitesimal isometry | 298 |

Part II Analysis of mechanical control systems

| | | |
|----------|--|-----|
| 6 | Stability | 313 |
| 6.1 | An overview of stability theory for dynamical systems | 315 |
| 6.1.1 | Stability notions | 315 |
| 6.1.2 | Linearization and linear stability analysis | 317 |
| 6.1.3 | Lyapunov Stability Criteria and LaSalle Invariance Principle | 319 |
| 6.1.4 | Elements of Morse theory | 325 |
| 6.1.5 | Exponential convergence | 327 |
| 6.1.6 | Quadratic functions | 329 |
| 6.2 | Stability analysis for equilibrium configurations of mechanical systems | 331 |
| 6.2.1 | Linearization of simple mechanical systems | 331 |
| 6.2.2 | Linear stability analysis for unforced systems | 334 |
| 6.2.3 | Linear stability analysis for systems subject to Rayleigh dissipation | 336 |
| 6.2.4 | Lyapunov stability analysis | 340 |

| | | |
|----------|---|------------|
| 6.2.5 | Global stability analysis | 344 |
| 6.2.6 | Examples illustrating configuration stability results | 345 |
| 6.3 | Relative equilibria and their stability | 349 |
| 6.3.1 | Existence and stability definitions | 349 |
| 6.3.2 | Lyapunov stability analysis | 351 |
| 6.3.3 | Examples illustrating existence and stability of relative equilibria | 355 |
| 6.3.4 | Relative equilibria for simple mechanical systems on Lie groups | 357 |
| 7 | Controllability | 367 |
| 7.1 | An overview of controllability for control-affine systems | 368 |
| 7.1.1 | Reachable sets | 369 |
| 7.1.2 | Notions of controllability | 371 |
| 7.1.3 | The Sussmann and Jurdjevic theory of attainability | 372 |
| 7.1.4 | From attainability to accessibility | 374 |
| 7.1.5 | Some results on small-time local controllability | 377 |
| 7.2 | Controllability definitions for mechanical control systems | 387 |
| 7.3 | Controllability results for mechanical control systems | 389 |
| 7.3.1 | Linearization results | 390 |
| 7.3.2 | Accessibility of affine connection control systems | 392 |
| 7.3.3 | Controllability of affine connection control systems | 394 |
| 7.4 | Examples illustrating controllability results | 398 |
| 7.4.1 | Robotic leg | 398 |
| 7.4.2 | Planar body with variable-direction thruster | 400 |
| 7.4.3 | Rolling disk | 402 |
| 8 | Low-order controllability and kinematic reduction | 411 |
| 8.1 | Vector-valued quadratic forms | 412 |
| 8.1.1 | Basic definitions and properties | 412 |
| 8.1.2 | Vector-valued quadratic forms and affine connection control systems | 414 |
| 8.2 | Low-order controllability results | 415 |
| 8.2.1 | Constructions concerning vanishing input vector fields | 416 |
| 8.2.2 | First-order controllability results | 417 |
| 8.2.3 | Examples and discussion | 420 |
| 8.3 | Reductions of affine connection control systems | 422 |
| 8.3.1 | Inputs for dynamic and kinematic systems | 422 |
| 8.3.2 | Kinematic reductions | 424 |
| 8.3.3 | Maximally reducible systems | 429 |
| 8.4 | The relationship between controllability and kinematic controllability | 432 |
| 8.4.1 | Implications | 433 |
| 8.4.2 | Counterexamples | 434 |

| | | |
|----------|--|-----|
| 9 | Perturbation analysis | 441 |
| 9.1 | An overview of averaging theory for oscillatory control systems | 442 |
| 9.1.1 | Iterated integrals and their averages | 443 |
| 9.1.2 | Norms for objects defined on complex neighborhoods . . | 446 |
| 9.1.3 | The variation of constants formula | 447 |
| 9.1.4 | First-order averaging | 451 |
| 9.1.5 | Averaging of systems subject to oscillatory inputs | 454 |
| 9.1.6 | Series expansion results for averaging | 459 |
| 9.2 | Averaging of affine connection systems subject to oscillatory controls | 463 |
| 9.2.1 | The homogeneity properties of affine connection control systems | 463 |
| 9.2.2 | Flows for homogeneous vector fields | 466 |
| 9.2.3 | Averaging analysis | 466 |
| 9.2.4 | Simple mechanical control systems with potential control forces | 471 |
| 9.3 | A series expansion for a controlled trajectory from rest | 473 |

Part III A sampling of design methodologies

| | | |
|-----------|---|-----|
| 10 | Linear and nonlinear potential shaping for stabilization | 481 |
| 10.1 | An overview of stabilization | 482 |
| 10.1.1 | Defining the problem | 483 |
| 10.1.2 | Stabilization using linearization | 485 |
| 10.1.3 | The gaps in linear stabilization theory | 487 |
| 10.1.4 | Control-Lyapunov functions | 489 |
| 10.1.5 | Lyapunov-based dissipative control | 490 |
| 10.2 | Stabilization problems for mechanical systems | 493 |
| 10.3 | Stabilization using linear potential shaping | 495 |
| 10.3.1 | Linear PD control | 495 |
| 10.3.2 | Stabilization using linear PD control | 497 |
| 10.3.3 | Implementing linear control laws on nonlinear systems . | 501 |
| 10.3.4 | Application to the two-link manipulator | 505 |
| 10.4 | Stabilization using nonlinear potential shaping | 507 |
| 10.4.1 | Nonlinear PD control and potential energy shaping | 507 |
| 10.4.2 | Stabilization using nonlinear PD control | 509 |
| 10.4.3 | A mathematical example | 515 |
| 10.5 | Notes on stabilization of mechanical systems | 515 |
| 10.5.1 | General linear techniques | 516 |
| 10.5.2 | Feedback linearization and partial feedback linearization | 517 |
| 10.5.3 | Backstepping | 517 |
| 10.5.4 | Passivity-based methods | 518 |
| 10.5.5 | Sliding mode control | 518 |
| 10.5.6 | Total energy shaping methods | 519 |

10.5.7 When stabilization by smooth feedback is not possible . 520

11 Stabilization and tracking for fully actuated systems 529

11.1 Configuration stabilization for fully actuated systems 530

11.1.1 Stabilization via configuration error functions 530

11.1.2 PD control for a point mass in three-dimensional
Euclidean space 532

11.1.3 PD control for the spherical pendulum 533

11.2 Trajectory tracking for fully actuated systems 534

11.2.1 Time-dependent feedback control and the tracking
problem 534

11.2.2 Tracking error functions 535

11.2.3 Transport maps 536

11.2.4 Velocity error curves 538

11.2.5 Proportional-derivative and feedforward control 540

11.3 Examples illustrating trajectory tracking results 542

11.3.1 PD and feedforward control for a point mass in
three-dimensional Euclidean space 542

11.3.2 PD and feedforward control for the spherical pendulum 543

11.4 Stabilization and tracking on Lie groups 546

11.4.1 PD control on Lie groups 547

11.4.2 PD and feedforward control on Lie groups 548

11.4.3 The attitude tracking problem for a fully actuated
rigid body fixed at a point 552

12 Stabilization and tracking using oscillatory controls 559

12.1 The design of oscillatory controls 560

12.1.1 The averaging operator 560

12.1.2 Inverting the averaging operator 563

12.2 Stabilization via oscillatory controls 567

12.2.1 Stabilization with the controllability assumption 568

12.2.2 Stabilization without the controllability assumption 571

12.3 Tracking via oscillatory controls 574

13 Motion planning for underactuated systems 583

13.1 Motion planning for driftless systems 584

13.1.1 Definitions 584

13.1.2 A brief literature survey of synthesis methods 587

13.2 Motion planning for mechanical systems 589

13.2.1 Definitions 589

13.2.2 Kinematically controllable systems 590

13.2.3 Maximally reducible systems 591

13.3 Motion planning for two simple systems 593

13.3.1 Motion planning for the planar rigid body 593

13.3.2 Motion planning for the robotic leg 596

| | | |
|----------|---|-----|
| 13.4 | Motion planning for the snakeboard | 598 |
| 13.4.1 | Modeling | 598 |
| 13.4.2 | Motion planning on $SE(2)$ for the snakeboard | 605 |
| 13.4.3 | Simulations | 612 |
| A | Time-dependent vector fields | 619 |
| A.1 | Measure and integration | 619 |
| A.1.1 | General measure theory | 619 |
| A.1.2 | Lebesgue measure | 621 |
| A.1.3 | Lebesgue integration | 622 |
| A.2 | Vector fields with measurable time-dependence | 624 |
| A.2.1 | Carathéodory sections of vector bundles and bundle maps | 624 |
| A.2.2 | The time-dependent Flow Box Theorem | 625 |
| B | Some proofs | 627 |
| B.1 | Proof of Theorem 4.38 | 627 |
| B.2 | Proof of Theorem 7.36 | 629 |
| B.3 | Proof of Lemma 8.4 | 635 |
| B.4 | Proof of Theorem 9.38 | 638 |
| B.5 | Proof of Theorem 11.19 | 648 |
| B.6 | Proof of Theorem 11.29 | 652 |
| B.7 | Proof of Proposition 12.9 | 654 |
| | References | 657 |
| | Symbol index | 689 |
| | Subject index | 705 |



<http://www.springer.com/978-0-387-22195-3>

Geometric Control of Mechanical Systems
Modeling, Analysis, and Design for Simple Mechanical
Control Systems

Bullo, F.; Lewis, A.D.

2005, XXIV, 727 p., Hardcover

ISBN: 978-0-387-22195-3