

Preface to the First Edition

It is said that Ramanujan taught himself mathematics by systematically working through 6000 problems¹ of Carr's *Synopsis of Elementary Results in Pure and Applied Mathematics*. Freeman Dyson in his *Disturbing the Universe* describes the mathematical days of his youth when he spent his summer months working through hundreds of problems in differential equations. If we look back at our own mathematical development, we can certify that problem solving plays an important role in the training of the research mind. In fact, it would not be an exaggeration to say that the ability to do research is essentially the art of asking the "right" questions. I suppose Pólya summarized this in his famous dictum: if you can't solve a problem, then there is an easier problem you can't solve – find it!

This book is a collection of about 500 problems in algebraic number theory. They are systematically arranged to reveal the evolution of concepts and ideas of the subject. All of the problems are completely solved and no doubt, the solutions may not all be the "optimal" ones. However, we feel that the exposition facilitates independent study. Indeed, any student with the usual background of undergraduate algebra should be able to work through these problems on his/her own. It is our conviction that the knowledge gained by such a journey is more valuable than an abstract "Bourbaki-style" treatment of the subject.

How does one do research? This is a question that is on the mind of every graduate student. It is best answered by quoting Pólya and Szegő: "General rules which could prescribe in detail the most useful discipline of thought are not known to us. Even if such rules could be formulated, they would not be very useful. Rather than knowing the correct rules of thought theoretically, one must have them assimilated into one's flesh and blood ready for instant and instinctive use. Therefore, for the schooling of one's powers of thought only the practice of thinking is really useful. The

¹Actually, Carr's *Synopsis* is not a problem book. It is a collection of theorems used by students to prepare themselves for the Cambridge Tripos. Ramanujan made it famous by using it as a problem book.

independent solving of challenging problems will aid the reader far more than aphorisms.”

Asking how one does mathematical research is like asking how a composer creates a masterpiece. No one really knows. However, it is clear that some preparation, some form of training, is essential for it. Jacques Hadamard, in his book *The Mathematician's Mind*, proposes four stages in the process of creation: preparation, incubation, illumination, and verification. The preparation is the conscious attention and hard work on a problem. This conscious attention sets in motion an unconscious mechanism that searches for a solution. Henri Poincaré compared ideas to atoms that are set in motion by continued thought. The dance of these ideas in the crucible of the mind leads to certain “stable combinations” that give rise to flashes of illumination, which is the third stage. Finally, one must verify the flash of insight, formulate it precisely, and subject it to the standards of mathematical rigor.

This book arose when a student approached me for a reading course on algebraic number theory. I had been thinking of writing a problem book on algebraic number theory and I took the occasion to carry out an experiment. I told the student to round up more students who may be interested and so she recruited eight more. Each student would be responsible for one chapter of the book. I lectured for about an hour a week stating and sketching the solution of each problem. The student was then to fill in the details, add ten more problems and solutions, and then typeset it into \TeX . Chapters 1 to 8 arose in this fashion. Chapters 9 and 10 as well as the supplementary problems were added afterward by the instructor.

Some of these problems are easy and straightforward. Some of them are difficult. However, they have been arranged with a didactic purpose. It is hoped that the book is suitable for independent study. From this perspective, the book can be described as a first course in algebraic number theory and can be completed in one semester.

Our approach in this book is based on the principle that questions focus the mind. Indeed, quest and question are cognates. In our quest for truth, for understanding, we seem to have only one method. That is the Socratic method of asking questions and then refining them. Grappling with such problems and questions, the mind is strengthened. It is this exercise of the mind that is the goal of this book, its *raison d'être*. If even one individual benefits from our endeavor, we will feel amply rewarded.

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