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Glossary of Symbols

A.P. arithmetic progression, $a, a + d, \ldots a + kd, \ldots$  

$a_1 \equiv a_2 \mod b$ $a_1$ congruent to $a_2$, modulo $b$;  
$a_1 - a_2$ divisible by $b$.  

$A(x)$ number of members of a sequence not exceeding $x$;  
e.g. number of amicable numbers not exceeding $x$  

$c$ a positive constant  
(not always the same!)  

$d_n$ difference between consecutive primes;  
$p_{n+1} - p_n$  

$d(n)$ the number of (positive) divisors of $n$; $\sigma_0(n)$  

$d \mid n$ $d$ divides $n$; $n$ is a multiple of $d$; there is an integer $q$  
such that $dq = n$  

$d \nmid n$ $d$ does not divide $n$  

$e$ base of natural logarithms;  
$2.718281828459045 \ldots$  

$E_n$ Euler numbers; coefficients in series for $\sec x$  

$\exp\{..\}$ exponential function  

$F_n$ Fermat numbers; $2^{2^n} + 1$