

# Preface

In 1977 the Mathematics Department at the University of California, Berkeley, instituted a written examination as one of the first major requirements toward the Ph.D. degree in Mathematics. This examination replaced a system of standardized Qualifying Exams. Its purpose was to determine whether first-year students in the Ph.D. program had mastered basic mathematics well enough to continue in the program with a reasonable chance of success.

Historically, any one examination is passed by approximately half of the students taking it and students are allowed three attempts. Since its inception, the exam has become a major hurdle to overcome in the pursuit of the degree and, therefore, a measure of the minimum requirements to successful completion of the program at Berkeley. Even though students are allowed three attempts, most would agree that the ideal time to complete the requirement is during the first month of the program rather than in the middle or end of the first year. This book was conceived on this premise, and its intent is to publicize the material and aid in the preparation for the examination during the undergraduate years, when one is deeply involved with the material that it covers.

The examination is now offered twice a year in the second week of each semester, and consists of 6 hours of written work given over a 2-day period with 9 problems each (10 before 1988). Students select 6 of the 9 problems (7 of 10 before 1988). Most of the examination covers material, mainly in analysis and algebra, that should be a part of a well-prepared mathematics student's undergraduate training. This book is a compilation of the more than 1000 problems which have appeared on the Prelims during the last few decades and currently make up a collection which is a delightful field to plow through, and solutions to most of them.

When Berkeley was on the Quarter system, exams were given three times a year: Spring, Summer, and Fall. Since 1986, the exams have been given twice a year, in January and September.

From the first examination through Fall 1981, the policy was: two attempts allowed; each examination 6 hours; total 14/20 problems. From Winter 1982 through Spring 1988, the policy was: two attempts allowed; each examination 8 hours; total 14/20 problems. Starting Fall 1988, the policy was: three attempts allowed; each examination 6 hours; total 12/18 problems. In all cases, the examination must be passed within 13 months of entering the Ph.D. program.

The problems are organized by subject and ordered in increasing level of difficulty, within clusters. Each one is tagged with the academic term of the exam in which it appeared using abbreviations of the type **Fa87** to designate the exam given in the **Fall** semester of **1987**. Problems that have appeared more than once have been merged and show multiple tags for each exam. Sometimes the merge required slight modifications in the text (a few to make the problem correct!), but the original text has been preserved in an electronic version of the exams (see Appendix A). Other items in the Appendices include the syllabus, passing scores for the exams and a Bibliography used throughout the solutions.

Classifying a collection of problems as vast as this one by subjects is not an easy task. Some of the problems are interdisciplinary and some have solutions as varied as Analysis and Number Theory (1.1.18 comes to mind!), and the choices are invariably hard. In most of these cases, we provide the reader with an alternative classification or pointers to similar problems elsewhere.

We would like to hear about other solutions to the problems here and comments on the existing ones. They can be sent by e-mail to the authors.

This project started many years ago, when one of us (PNdS) came to Berkeley and had to go through the lack of information and uncertainties of the exam and got involved with a problem solving group. First thanks go to the group's members: Dino Lorenzini, Hung The Dinh, Kin-Yin Li, and Jorge Zubelli, and then to the Prelim Workshop leaders, many of whose names escape us now but the list includes, besides ourselves, Matthew Wiener, Dmitry Gokhman, Keith Kearnes, Geon Ho Choe, Mike May, Eliza Sachs, Ben Lotto, Ted Jones, David Cruz-Uribe, Jonathan Walden, Saul Schleimer and Howard Thompson; and also to the many people we have discussed these problems with like Don Sarason, George Bergman, Reginald Koo, D. Popa, C. Costara, József Sándor, Elton Hsu, Enlin Pan, Bjorn Poonen, Assaf Wool and Jin-Gen Yang. Many thanks to Debbie Craig for swift typesetting of many of the problems and to Janet Yonan for her help with the archeological work of finding many of the old and lost problem sets, and finally to Nefeli's for the best coffee west of Rome, we would not have survived without it!

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This is a project that could not have been accomplished in any typesetting system other than  $\text{\TeX}$ . The problems and solutions are part of a two-pronged database that is called by sourcing programs that generate several versions (working, final paper version, per-exams list, and the on-line HTML and PDF versions) from a single source. Silvio Levy's  $\text{\TeX}$  support and counseling was a major resource backing our efforts and many thanks also to Noam Shomron for help with non-standard typesetting.

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