More than seventy years ago Michael Goldberg showed that a fullerene polyhedron is mathematically possible for polyhedra with \( n \) atoms, where \( n=20+2k \) for integer values of \( k\geq 0 \), with the sole exception of \( k=1 \) (the nonexistent 12-pentagon and 1-hexagon 22-atom cage—the \( C_{22} \) fullerene) [1, 2]. This was later proved more rigorously by Grünbaum and Motzkin [3], who showed that a planar 3-connected multi-\( k \)-gon (for \( k=3, 4, 5 \)) with only one face being different from the other \( k \)-gons is not possible. In the fullerene context, this statement means that it is not possible to have a fullerene with all pentagons aside from just one hexagon. Therefore, \( C_{22} \) is the forbidden fullerene. Experimentation with fullerene graphs (planar projections) reveals that it is indeed impossible to construct a 22-vertex polyhedron with 12 pentagons and only one hexagon. Figure 1 (left) shows the projection of the smallest fullerene (\( C_{20} \)), consisting of only 12 pentagons. An attempt to construct a fullerene with only one hexagon at the center (Fig. 1 right) fails because the final (encompassing) face of the graph must be another hexagon, thus resulting in a polyhedron with a total of two hexagons and 24 vertices (\( C_{24} \) fullerene) [4].

Similar geometric restrictions exist for chemically stable fullerenes with all pentagon faces isolated from each other. The smallest of these fullerenes is the famous \( C_{60} \), followed by \( C_{70} \) and then by any other even number such as \( C_{72} \), \( C_{74} \), etc. \( C_{62}, C_{64}, C_{66} \) and \( C_{68} \) fullerenes where all of the pentagon faces are isolated from each other (surrounded by hexagons) are nonexistent.

References

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