

# Theory and Applications of Computability

Springer book series in cooperation with the association  
**Computability in Europe**

## Series Overview

Books published in this series will be of interest to the research community and graduate students, with a unique focus on issues of computability. The perspective of the series is multidisciplinary, recapturing the spirit of Turing by linking theoretical and real-world concerns from computer science, mathematics, biology, physics, and the philosophy of science.

The series includes research monographs, advanced and graduate texts, and books that offer an original and informative view of computability and computational paradigms.

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## Series Information

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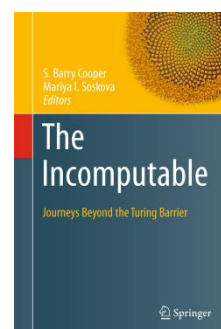
## Latest Book in Series

Title: **The Incomputable – Journeys Beyond the Turing Barrier**

Authors: S. Barry Cooper, Mariya Soskova

Notes: ISBN 978-3-319-43667-8, publication 05/17

This book questions the relevance of computation to the physical universe. Our theories deliver computational descriptions, but the gaps and discontinuities in our grasp suggest a need for continued discourse between researchers from different disciplines, and this book is unique in its focus on the mathematical theory of incomputability and its relevance for the real world. The core of the book consists of thirteen chapters in five parts on extended models of computation; the search for natural examples of incomputable objects; mind, matter, and computation; the nature of information, complexity, and randomness; and the mathematics of emergence and morphogenesis. This book will be of interest to researchers in the areas of theoretical computer science, mathematical logic, and philosophy.



## All Books in Series

**Title: Turing Computability – Theory and Applications**

Authors: Robert I. Soare

Notes: ISBN 978-3-642-31932-7, publication 2016

Turing's famous 1936 paper introduced a formal definition of a computing machine, a Turing machine. This model led to both the development of actual computers and to computability theory, the study of what machines can and cannot compute. This book presents classical computability theory from Turing and Post to current results and methods, and their use in studying the information content of algebraic structures, models, and their relation to Peano arithmetic. The author presents the subject as an art to be practiced, and an art in the aesthetic sense of inherent beauty which all mathematicians recognize in their subject. Part I gives a thorough development of the foundations of computability, from the definition of Turing machines up to finite injury priority arguments. Key topics include relative computability, and computably enumerable sets, those which can be effectively listed but not necessarily effectively decided, such as the theorems of Peano arithmetic. Part II includes the study of computably open and closed sets of reals and basis and nonbasis theorems for effectively closed sets. Part III covers minimal Turing degrees. Part IV is an introduction to games and their use in proving theorems. Finally, Part V offers a short history of computability theory.

**Title: Higher-Order Computability**

Authors: John Longley, Dag Norman

Notes: ISBN 978-3-662-47991-9, publication 2015

This book offers a self-contained exposition of the theory of computability in a higher-order context, where 'computable operations' may themselves be passed as arguments to other computable operations. The subject originated in the 1950s with the work of Kleene, Kreisel and others, and has since expanded in many different directions under the influence of workers from both mathematical logic and computer science. The ideas of higher-order computability have proved valuable both for elucidating the constructive content of logical systems, and for investigating the expressive power of various higher-order programming languages. In contrast to the well-known situation for first-order functions, it turns out that at higher types there are several different notions of computability competing for our attention, and each of these has given rise to its own strand of research. In this book, the authors offer an integrated treatment that draws together many of these strands within a unifying framework, revealing not only the range of possible computability concepts but the relationships between them. The book will serve as an ideal introduction to the field for beginning graduate students, as well as a reference for advanced researchers.

**Title: Apartness and Uniformity – A Constructive Development**

Authors: Douglas S. Bridges, Luminita Simona Viță

Notes: ISBN 978-3-642-22414-0, publication 2011

Largely an exposition of the authors' own research, this is the first book dealing with the apartness approach to constructive topology, and is a valuable addition to the literature on constructive mathematics and on topology in computer science. It is aimed at graduate students and advanced researchers in theoretical computer science, mathematics, and logic who are interested in constructive/algorithmic aspects of topology.

**Title: Algorithmic Randomness and Complexity**

Authors: Rodney G. Downey, Denis R. Hirschfeldt

Notes: ISBN 978-0-387-95567-4, publication 2010

The theory of algorithmic randomness uses tools from computability theory and algorithmic information theory to address questions such as these. Much of this theory can be seen as exploring the relationships between three fundamental concepts: relative computability, as measured by notions such as Turing reducibility; information content, as measured by notions such as Kolmogorov complexity; and randomness of individual objects, as first successfully defined by Martin-Löf. Although algorithmic randomness has been studied for several decades, a dramatic upsurge of interest in the area, starting in the late 1990s, has led to significant advances. This is the first comprehensive treatment of this important field, designed to be both a reference tool for experts and a guide for newcomers. It surveys a broad section of work in the area, and presents most of its major results and techniques in depth. Its organization is designed to guide the reader through this large body of work, providing context for its many concepts and theorems, discussing their significance, and highlighting their interactions. It includes a discussion of effective dimension, which allows us to assign concepts like Hausdorff dimension to individual reals, and a focused but detailed introduction to computability theory. It will be of interest to researchers and students in computability theory, algorithmic information theory, and theoretical computer science.



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