Chapter 2
Lake-Catchment System
and Drainage System

The drainage system is a dominant structure of catchments. Studies on drainage systems started in the 1940s and were related to the development of quantitative geomorphology in the USA. Horton (1945) was a pioneer in this area. He and his successors developed ordering systems for the drainage structure (Scheidegger 1965; Fig. 2.1), which was linked to the development of the theory of fractal (Mandelbrot 1977).

Considering the temporal and spatial development of a drainage system, and the continuous records of various climatic regimes at various scales, the following are discussed; (1) the fractal structure of the drainage system in a catchment, and (2) the compatibility of drainage systems in various zones (“ergodic” reasoning).

The fractal structure of the drainage system in a catchment is more understandable in the self-similarity between small-scale systems and large-scale ones; “cyclicity” in scales is of great importance for structural development in the system. A typical structure of drainage system is shown in Figs. 2.2 and 2.3 (Tokunaga 1978; Kashiwaya 1986). The cyclicity was first identified and described by Tokunaga (1978) (introduced by Tarboton 1996). The number of streams of each order $\omega$ within a basin of order $\Omega$ (Fig. 2.2) (generalization of Horton law) is given by;

$$N(\Omega, \omega) = \frac{Q^{\Omega-\omega-1} - P^{\Omega-\omega-1}}{Q-P} (2 + \varepsilon_{1} - P)Q + (2 + \varepsilon_{1})P^{\Omega-\omega-1}.$$

$P$ and $Q$ are, respectively, given by

$$P = \frac{2 + \varepsilon_{1} + K - \sqrt{(2 + \varepsilon_{1} + K)^2 - 8K}}{2}$$
and

\[ Q = \frac{2 + \varepsilon_1 + K + \sqrt{(2 + \varepsilon_1 + K)^2 - 8K}}{2}, \]

with \( \varepsilon_1 = \kappa^j e_{k-1} \) and \( K = (\kappa^j e_{k}/\kappa^j e_{k-1})^{1/(\kappa^j - 1)} \), where \( \kappa^j e_{\lambda} \) is the number of stream order \( \lambda \) entering a stream of order \( \kappa \) from the sides.

Fig. 2.1 Ordering method for streams proposed by a Horton (1945), b Strahler (1957), c Shreve (1966), and d Scheidegger (1965)
Fig. 2.2 Fractal structure of a generalized stream network (modified after Tokunaga 1978)

Fig. 2.3 Fractal structure of a small stream network (modified after Kashiwaya 1986)
Drainage systems in various stages under similar environmental conditions and structures are available for arranging a time sequence of a drainage system development; this is the “ergodic” reasoning in geomorphology (Paine 1985; Yatsu 1986). A mathematical model of the drainage system’s development has been previously introduced and validated with field data (Kashiwaya 1983, 1987). A basic idea of this model was given from a rill network study in a slope system (Kashiwaya 1979, 1980). As is shown in Fig. 2.4, temporal change in drainage density is expressed by

\[
\frac{dD(t)}{dt} = \beta'(t)\{1 - D(t)/\delta(t)\}D(t),
\]

where \(D(t)\) is the drainage density, \(\beta'(t)\) is the relative erosional force, and \(\delta(t)\) is the maximum drainage density determined by the physical properties of the catchment. It is solved, setting as an initial condition \(D(t) = D_0\) for \(t = t_0\)

\[
D(t) = e^{-\int_{t_0}^{t} \beta'(\tau) d\tau} \left[ \int_{t_0}^{t} \frac{\beta'(\tau)}{\delta(\tau)} e^{-\int_{t_0}^{\tau} \beta'(\xi) d\xi} d\tau + 1/D_0 \right].
\]

This equation provides the temporal change of the drainage density in a lake-catchment system. In general, we have no long-term field data of the drainage

![Fig. 2.4](image-url) An evolitional model of a stream network (modified after Kashiwaya 1986). Solid line: streams at time \(t\). Dotted line: elongated segment of streams at time \(t + \Delta t\)
density, erosional force, and physical properties related to the system. Therefore, using dated catchment systems (or geomorphic surfaces), we assume that the drainage density in the dated systems or surfaces corresponds to the development stage of a certain catchment system, which is a typical example of the “ergodic” reasoning applied to catchment systems. In a hypothetical system, it is assumed that the function of the relative erosional force acting on the \( i \)-th system is expressed as \( \beta_i'(t) \) and the river formation begins at \( t_i \). Then, the present drainage density is expressed as follows:

\[
D_1 (t_1, T) = e^{-\int_{t_1}^{T} \beta_i'(\tau) d\tau} \left[ \int_{t_1}^{T} (\beta_1'(\tau)/\delta_1(\tau)) e^{-\int_{\tau}^{T} \beta_i'(\xi) d\xi} d\tau + 1/D_0 \right]
\]

\[
D_2 (t_2, T) = e^{-\int_{t_2}^{T} \beta_i'(\tau) d\tau} \left[ \int_{t_2}^{T} (\beta_2'(\tau)/\delta_2(\tau)) e^{-\int_{\tau}^{T} \beta_i'(\xi) d\xi} d\tau + 1/D_0 \right], \quad (2.3)
\]

\[
D_i (t_i, T) = e^{-\int_{t_i}^{T} \beta_i'(\tau) d\tau} \left[ \int_{t_i}^{T} (\beta_i'(\tau)/\delta_i(\tau)) e^{-\int_{\tau}^{T} \beta_i'(\xi) d\xi} d\tau + 1/D_0 \right], \quad (2.4)
\]

where \( D(t_i, T) \), \( \beta_i'(t) \), \( \delta_i(t) \), and \( D_{0i} \) are the drainage density, erosional force, maximum drainage density, and initial drainage density of the \( i \)-th system (geomorphic surface), respectively. If the relative erosional forces act equally on all systems (nearly under the same environmental (climatic) conditions) and the physical properties of the systems related to the erosional forces are nearly the same, we obtain

\[
\beta_1'(t) = \beta_2'(t) = \cdots = \beta_i'(t) = \cdots = \beta'(t)
\]

\[
\delta_1(t) = \delta_2(t) = \cdots = \delta_i(t) = \cdots = \delta(t)
\]

\[
D_{01} = D_{02} = \cdots = D_{0i} = \cdots = D_0.
\]

Then, the drainage density of a series of systems during \( T - t \) can be expressed in the same form:

\[
D(t, T) = e^{-\int_{t}^{T} \beta_i'(\tau) d\tau} \left[ \int_{t}^{T} \left( \beta'(\tau)/\delta(\tau) \right) e^{-\int_{\tau}^{T} \beta'(\xi) d\xi} d\tau + 1/D_0 \right]. \quad (2.5)
\]

It is necessary to give the functions \( \beta'(t) \) and \( \delta(t) \), \( D_0 \) in the equation and field data to evaluate this equation. However, establishing the functions and obtaining dated field data are difficult. Thus, it is assumed, as a first approximation, that \( \beta'(t) \) and \( \delta(t) \) are constant for this system:
This equation can be evaluated by using dated field data. Two examples are introduced: the drainage densities measured in terrace surfaces formed at different ages in Kanto, Japan (Kashiwaya 1983; Fig. 2.5a), and the drainage densities in drift surfaces formed at different times in North America (Kashiwaya 1987; Fig. 2.5b; Ruhe 1952). These support that the model introduced here is plausible,
although it is difficult to establish the functions and the physical properties related to $D_{oi}$ (Fig. 2.4). However, knowledge of functions relating to external forces, weathering conditions, and the physical properties of materials constituting landforms ($\beta'(t)$, $\delta(t)$, $D_0$) is fundamental not only in limnogeomorphology, but also in geomorphology in general. Thus, sustainable efforts to obtain such knowledge are required.

References

Mandelbrot BB (1977) Fractals: form, change and dimension. Freeman, San Francisco, p 365
Scheidegger AE (1965) The algebra of stream-order numbers. US Geol Surv Prof Pap 525B:187–189
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