Chapter 2
Simple Interest and Discount

Abstract Chapter 2 contains basic formulas for simple interest and discount: 2.1. Simple Interest, 2.2. Calendar Conventions, 2.3. Simple Interest with Principals Credited *m*thly, 2.4. Simple Discount.

2.1 Simple Interest

- Interest (1) from the point of view of *debtor*: is the price paid for borrowed money; (2) from the point of view of *creditor*: is the reward for postponed consumption and uncertainty of investment; the interests can be of various types:
  - *active*: are interests paid by clients for credits (loans) from banks
  - *effective*: are actual interests respecting various effects that influence returns (due to conversions m times per year, time shifts, charges, and the like)
  - *nominal*: are quoted interests that do not respect the actual conversion periods
  - *passive*: are interests paid by banks for deposits from clients
  - *real*: are interests adjusted for inflation
  - *returns*: are yields (gains, profits) due to investment
- *Simple interest* model: means that a loan has an interest calculated at any period entirely from the original principal so that the amount due increases linearly

Denotation:

i	annual (p.a.) interest rate given as a decimal value
p	annual (p.a.) interest rate given as a per cent value: $i = p/100$
t	time measured in years (e.g. $t = 0.5$ means half a year)
k	time measured in days: $t = k/365$ (see Sect. 2.2)
I_t	interest credited at time t
P	*principal* (principal capital, *present value*)
S_t	*amount due* at time t (*future value*, terminal value): $S_0 = P$

T. Cipra, *Financial and Insurance Formulas*, DOI 10.1007/978-3-7908-2593-0_2,

$$I_t = Pit = Pi\frac{k}{365} = P\frac{p}{100}\frac{k}{365}$$

(*simple interest* credited at time t: it is applied when the time t does not exceed 1 year, i.e. $0 \le t \le 1$ or $0 \le k \le 365$)

$$P = \frac{I_t}{it} \quad (\textit{principal}\, P)$$

$$i = \frac{I_t}{Pt} \quad (\textit{interest rate}\, i); \qquad p = \frac{100 I_t}{Pt} \quad (\textit{interest rate}\, p \text{ given as percent})$$

$$t = \frac{I_t}{Pi} \quad (\textit{time}\, t); \qquad k = \frac{365 I_t}{Pi} \quad (\textit{time}\, k \text{ measured in days})$$

$$I = \frac{\mathrm{IN}_1 + \cdots + \mathrm{IN}_n}{\mathrm{ID}}$$

where $\mathrm{IN}_j = (P_j k_j)/100$ is *interest number* for principal P_j and time k_j measured in days ($j = 1, \ldots, n$); $\mathrm{ID} = 365/p$ is *interest divisor* for interest rate p given as per cent

(*simple interest for checking account* (*demand deposit*): principals $P_1, \ldots, P_n$ bear interests within $k_1, \ldots, k_n$ days, respectively, due to a fixed interest rate p)

$$S_t = P + I_t = P(1 + it) = P\left(1 + i\frac{k}{365}\right) = P\left(1 + \frac{p}{100}\frac{k}{365}\right)$$

(*amount due* at time t: is the principal P with simple interest I_t accrued up to time t; it is applied when the time t does not exceed 1 year, i.e. $0 \le t \le 1$ or $0 \le k \le 365$)

$$P = \frac{S_t}{1 + it} \quad (\textit{principal}\, P)$$

$$i = \frac{S_t - P}{Pt} \quad (\textit{interest rate}\, i); \qquad p = \frac{100(S_t - P)}{Pt} \quad (\textit{interest rate}\, p \text{ given as percent})$$

$$t = \frac{S_t - P}{Pi} \quad (\textit{time}\, t \text{ in years}); \qquad k = \frac{365(S_t - P)}{Pi} \quad (\textit{time}\, k \text{ measured in days})$$

$$\bar{t} = \frac{S_1 + \cdots + S_n - P}{Pi}, \quad \text{where } P = \frac{S_1}{1 + it_1} + \cdots + \frac{S_n}{1 + it_n}$$

(*mean pay-off time* for simple interest: is an equivalent time at which all amounts $S_1, \ldots, S_n$ corresponding originally to times $t_1, \ldots, t_n$ could be paid off all at once)

- *Discount rate*: is the interest rate charged on discount loans (short-term funds) by the central bank to commercial banks (such loans provide reserves to banks in a time of need and are a tool of monetary policy: e.g. the central bank increases the discount rate when a higher inflation is expected)
- *Repo rate*: is the interest rate charged by the central bank when purchasing bills of exchange discounted by commercial banks (more generally, repo rates are the rates applied in any repo operation)
- *Interbank interest rates*: are the interest rates for short-term loans among commercial banks (their motivation is the same as in the case of discount rate); e.g. LIBOR (London Interbank Offered Rate) and LIBID (London Interbank Bid Rate) are published daily for leading currencies and various maturities as the trimmed average of eight or sixteen leading interest rates on the interbank market in UK; similarly, one applies FIBOR in Germany (Frankfurt) or EURIBOR in EU

2.2 Calendar Conventions

- *Calendar conventions*: are rules how to count the difference between two dates; in particular, one applies them in formulas of simple interest model (see Sect. 2.1) and simple discount model (see Sect. 2.4)

Denotation:

DMY symbol for date (e.g. the date November 6, 2009 corresponds to $D = 6$, $M = 11$, $Y = 2009$); for a time interval one needs dates $T_1 = D_1M_1R_1$ and $T_2 = D_2M_2R_2$

$$t = \frac{k}{360} \quad \text{or} \quad t = \frac{k}{365} \quad \text{or} \quad t = \frac{k}{\text{act}}$$

(*calendar conventions*: express the time t in interest or discount models as a fraction of year (see examples *thereinafter*); they differ according to countries and financial products)

$$t = \frac{360(R_2 - R_1) + 30(M_2 - M_1) + \min\{D_2;\ 30\} - \min\{D_1;\ 30\}}{360}$$

(*calendar Euro-30/360*: all months have 30 days and all years have 360 days)

$$t = \frac{360(R_2 - R_1) + 30(M_2 - M_1) + D_2^* - D_1^*}{360}$$

(*calendar US-30/360*: the asterisks mean that all dates ending on 31st are changed to 30th as for Euro-30/360 with the only exception, namely if $D_1 < 30$ and $D_2 = 31$ then one changes T_2 to the first day of the next month)

$$t = \frac{T_2 - T_1}{360}$$

(*calendar act/360*: uses the actual number of days of the given period (one denotes it as $T_2 - T_1$), but 360 days of the year are considered in denominator of the corresponding fraction; it is used e.g. in Germany for operations with eurocurrencies and for floaters)

$$t = \frac{T_2 - T_1}{365}$$

(*calendar act/365*: uses the actual number of days of the given period (one denotes it as $T_2 - T_1$) and 365 days of the year are considered in denominator (also for the leap year); it is used e.g. in UK or for short-termed securities on German money-market)

$$t = \frac{k_1}{\text{number of days in beginning year } R_1} + R_2 - R_1 - 1 + \frac{k_2}{\text{number of days in ending year } R_2}$$

(*calendar act/act*: uses the actual number of days of the given period and the actual number of days of particular years; k_1 is the actual number of days of the given period in the beginning year R_1 and k_2 is the actual number of days of the given period in the ending year R_2 (if $R_1 = R_2$, then k_1 is the actual number of days from the beginning of the given period till the end of this year and k_2 is the actual number of days from the beginning of this year till the end of the given period)

$$i_2 = i_1 \frac{t_1}{t_2}$$

(conversion of the rate of return i_1 to i_2 due to change from the calendar convention t_1 to t_2)

$$i_2 = i_1 \frac{365}{360}$$

(example of conversion of the rate of return i_1 to i_2 due to change from the calendar convention t_1 = act/360 to t_2 = act/365)

- Conventions that are applied when the maturity date is not the bank day:
  - *following* day: the maturity date is taken as the following bank day
  - *modified following* day: the maturity date is taken as the following bank day, if it still lies in the same month; otherwise one takes the preceding bank day
  - *preceding* day: the maturity date is taken as the preceding bank day
  - *modified preceding* day: the maturity date is taken as the preceding bank day, if it still lies in the same month; otherwise one takes the following bank day
  - *second-day-after*: the maturity date is taken as the following second bank day

2.3 Simple Interest with Principals Credited *m*thly

- *Simple interest with principals credited mthly*: instead of the usual calculation of the simple interest at the end of an annual period, one divides it to m subperiods (e.g. to months for $m = 12$) as if the same principals r were credited from the beginnings or from the ends of particular subperiods to the end of the given annual period; at this date a single *annual compensation* for all subperiods is paid up

Denotation:

i annual (p.a.) interest rate
r amounts of principals credited $m$thly (e.g. monthly for $m = 12$)
R annual compensation for all subperiods

$$R = rm + r\frac{i}{m}(m + (m-1) + \cdots + 1) = r\left(m + \frac{m+1}{2}i\right)$$

(simple interest with principals credited $m$thly from the *beginnings* of particular subperiods: e.g. for $m = 12$ one obtains $R = r\cdot(12 + 6.5\cdot i)$)

$$R = rm + r\frac{i}{m}((m-1) + (m-2) + \cdots + 1) = r\left(m + \frac{m-1}{2}i\right)$$

(simple interest with principals credited m-thly from the *ends* of particular subperiods: e.g. for $m = 12$ one obtains $R = r\cdot(12 + 5.5\cdot i)$)

2.4 Simple Discount

- *Simple discount*: is an interest transaction common mainly for short-term loan instruments, i.e. with maturity up to 1 year (bills of exchange, certificates of deposits (CD's), Treasury bills (T-bills), and the like), where the price of the corresponding loan is set down by subtracting the so-called *discount* from the amount due; such a loan makes use of the *discount principle*, i.e. the corresponding interest is credited at the beginning of the discount period (*interest-in-advance*), while in the simple interest model the interest is credited *in arrears* at the end of the interest period

Denotation:

d annual (p.a.) discount rate given as a decimal value
t time measured in years (e.g. $t = 0.5$ means half a year)
k time measured in days: $t = k/365$ (see Sect. 2.2)
D_t discount (interest-in-advance) credited at the beginning of discount period t

P *principal*
S_t *amount due* at time t

$$D_t = S_t dt = S_t d \frac{k}{365} \quad (\textit{discount} \text{ credited at the beginning of discount period } t)$$

$$P = S_t - D_t = S_t(1 - dt) = S_t\left(1 - d\frac{k}{365}\right) \; (\textit{principal } P)$$

$$d = \frac{S_t - P}{S_t t} \; (\textit{discount rate } d)$$

$$t = \frac{S_t - P}{S_t d} \; (\textit{discount period } t \text{ measured in years})$$

$$k = \frac{365(S_t - P)}{S_t d} \; (\textit{discount period } k \text{ measured in days})$$

$$i = \frac{S_t - P}{Pt} \quad > \quad d = \frac{S_t - P}{S_t t} \; (\text{comparison of interest rate } i \text{ and discount rate } d)$$

$$i = \frac{d}{1 - dt}$$

(relation between interest rate i and discount rate d)

Further Reading

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