# Contents

## 1 Modelling with Graphs

1.1 Actions, Events, Programs .................................. 2  
1.2 Time ...................................................................... 6  
1.3 Knowledge and Belief ............................................. 7  
1.4 Obligations and Permissions ..................................... 12  
1.5 Relations Between Objects ....................................... 13  
1.6 Kripke Models: The Formal Definition ....................... 14  
1.7 Kripke Models: Building Them with LoTREC .............. 16  
1.8 The Typewriter Font Convention ............................... 18  
1.9 Modelling Theories by Classes of Kripke Models .......... 19  
1.10 Summary .......................................................... 20

## 2 Talking About Graphs

2.1 The Formal Language .............................................. 25  
2.1.1 Atomic Formulas ................................................ 26  
2.1.2 Boolean Connectives ......................................... 26  
2.1.3 Modal Connectives ............................................. 27  
2.1.4 Duality of Modal Connectives ............................... 29  
2.1.5 The Definition of Formulas .................................... 30  
2.1.6 Analysing a Formula: Subformulas, Formula Length,  
     Arity .................................................................. 31  
2.1.7 Parenthesis Conventions ...................................... 33  
2.2 Syntax Declaration in LoTREC ................................. 34  
2.2.1 Prefix Notation .................................................. 35  
2.2.2 Defining Atomic Labels ..................................... 35  
2.2.3 Defining Connectives ......................................... 35  
2.2.4 Complex Labels in LoTREC ................................. 37  
2.2.5 Displaying Formulas .......................................... 38  
2.3 Truth Conditions .................................................... 39  
2.4 How to Do Model Checking in LoTREC ..................... 41  
2.5 Modal Logics and Reasoning Problems ....................... 43
2.6 Modal Logics and Their Axiomatisations 46
2.7 A Note on Computational Complexity 50
2.8 Summary 50

3 The Basics of the Model Construction Method 53
3.1 Definition of Labelled Graphs 54
3.2 Building an Example Model by Hand 55
3.2.1 The Idea: Apply the Truth Conditions 55
3.2.2 Decomposing the Example Formula 56
3.2.3 Extracting a Kripke Model from a Premodel 59
3.3 How to Turn Truth Conditions into Tableau Rules 61
3.4 Tableaux: Some Fundamental Notions 62
3.5 The Language of Tableau Rules in LoTREC 64
3.6 Strategies: How to Combine LoTREC Rules 69
3.6.1 Rule Application in LoTREC: Everywhere and Simultaneously 70
3.6.2 Stopping and Killing Strategies 71
3.6.3 A Strategy for Any Formula: Saturation by Rule Iteration 72
3.6.4 Nesting Strategies 74
3.6.5 Prioritising Rule Application 75
3.7 Exercises: Adding Connectives 77
3.8 From Monomodal K to Multimodal Logic K_n 78
3.9 Description Logic ALC 78
3.10 Taming the Rule for Disjunction 79
3.10.1 Redundant Disjunctions 80
3.10.2 The Cut Rules 80
3.11 Soundness, Termination, and Completeness 84
3.12 Summary 85

4 Logics with Simple Constraints on Models 87
4.1 K_3 ⊕ Inclusion: Inclusion of Relations 89
4.2 KT: Reflexivity 91
4.3 KB: Symmetry 93
4.4 K.Alt_1: Partial Function 94
4.5 KD: Seriality 97
4.6 K.2: Confluence 101
4.7 S5: A Single Equivalence Relation 106
4.8 HL: Hybrid Logic Nominals 109
4.9 A General Termination Theorem 114
4.9.1 Execution Trace 115
4.9.2 Monotonic Rules 116
4.9.3 Size of Labels 116
4.9.4 Sublabels Modulo Negation 117
4.9.5 Strictly Analytic Rules, Connected Rules 118
4.9.6 The Theorem 119
4.9.7 Proof of the Termination Theorem .................. 120
4.10 Summary ........................................... 123

5 Logics with Transitive Accessibility Relations ............... 125
  5.1 K4: Transitivity ........................................ 125
  5.2 Marking Nodes and Expressions in LoTREC .................. 133
  5.3 Intuitionistic Logic: Reflexivity, Transitivity and Persistence ... 134
  5.4 GL: Transitivity and Noetherianity ....................... 139
  5.5 Completeness vs. Termination ............................ 141
  5.6 Another General Termination Theorem by Checking for Loops ... 143
  5.7 Other Logics as Exercises .............................. 145
  5.8 Summary ............................................. 146

6 Model Checking ............................................. 147
  6.1 Model Checking by Hand .................................... 147
  6.2 How to Implement Model Checking in LoTREC ................. 151
  6.3 Conclusion ............................................. 156

7 Modal Logics with Transitive Closure ......................... 157
  7.1 LTL: Linear-Time Temporal Logic ......................... 158
    7.1.1 Syntax and Semantics ............................... 159
    7.1.2 Model Construction for LTL in LoTREC: Saturating Premodels .... 161
    7.1.3 LoTREC Rules for LTL: Termination by Node-Inclusion Tests ........ 164
    7.1.4 LoTREC Rules for LTL: Fulfilment of Eventualities ...... 167
    7.1.5 LoTREC Rules for LTL: Termination by Node-Equality Test ........ 170
    7.1.6 LoTREC Strategy for LTL ........................... 172
  7.2 PDL: Propositional Dynamic Logic ......................... 173
    7.2.1 Syntax of PDL .................................... 174
    7.2.2 Semantics of PDL ................................ 175
    7.2.3 LoTREC Rules for PDL: Saturating Premodels ............ 176
    7.2.4 LoTREC Rules for PDL: Ensuring Termination ........... 179
    7.2.5 LoTREC Rules for PDL: Fulfilment of Eventualities .... 181
    7.2.6 LoTREC Strategy for PDL ........................... 186
  7.3 Conclusion ............................................. 189

References ..................................................... 191

Index .......................................................... 195
Kripke's Worlds
An Introduction to Modal Logics via Tableaux
Gasquet, O.; Herzig, A.; Said, B.; Schwarzentruber, F.
2014, XV, 198 p. 73 illus., Softcover
ISBN: 978-3-7643-8503-3
A product of Birkhäuser Basel