

Preface: What is Mathematics?

Günter M. Ziegler

This book is an Invitation to Mathematics.

But *What is Mathematics?* This is a question that asks us for a definition. You could look in *Wikipedia* and find the following:

Mathematics is the study of quantity, structure, space, and change. Mathematicians seek out patterns, formulate new conjectures, and establish truth by rigorous deduction from appropriately chosen axioms and definitions.

Quantity, structure, space, and change? These words outline a vast field of knowledge — and they are combined with a very narrow, mechanistic, and, frankly, quite boring description of “what mathematicians do”. Should “what mathematicians do” really be a part of the definition?

The definition given by the German *Wikipedia* is interesting in a different way: it stresses that there is no definition of mathematics, or at least no commonly accepted one. I translate:

Mathematics is the science that developed from the investigation of figures and computing with numbers. For mathematics, there is no commonly accepted definition; today it is usually described as a science that investigates abstract structures that it created itself for their properties and patterns.

Is this a good definition, a satisfactory answer to the question “What is Mathematics”? I believe that *Wikipedia* (in any language) does not give a satisfactory answer. At the same time, and much more importantly, high school curricula do not give a satisfactory answer. Even the famous book by Richard Courant and Herbert Robbins entitled “What is Mathematics?” (and subtitled “An Elementary Approach to Ideas and Methods”) does not give a satisfactory answer.

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Perhaps it is impossible to give a good definition in a sentence or two. Indeed, I claim that there cannot be one single answer that we could be content with: mathematics in the 21-st century is a huge body of knowledge and a very diverse area of study. There are thus so many ways to experience mathematics — the arenas of national and international competitions, and research experiences that range from years spent working in solitude (think of Andrew Wiles, who proved Fermat’s Last Theorem, or Grigori Perelman, who proved the Poincaré conjecture) to coffee break discussions at conferences to massive collaborations on internet platforms (such as the POLYMATH projects initiated by Michael Nielsen, Timothy Gowers, Terence Tao, and others).

But perhaps the English *Wikipedia* is right in one aspect — that in approaching the science called mathematics one should look at the people who do mathematics. So *what is mathematics as an experience?* What does it mean to *do* mathematics?

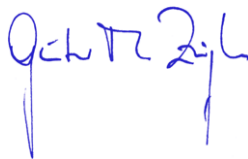
This book is an invitation to mathematics comprised of contributions by leading mathematicians. Many of them were initiated to mathematics, and led to mathematics research, through competitions such as the mathematical olympiads — one of the ways to get attracted to and drawn into mathematics. This book builds a link between the “domesticated” mathematics taught at high schools or used in competitions and the “wild” and “free” world of mathematical research. As a former high school student, successful participant at competitions such as the IMO 1981, and now professor of mathematics who is doing research and who is active in communicating mathematics to the public, I have personally experienced all these kinds of mathematics, and I am excited about this book and the link that it provides.

The starting point of this book was an event that I had the pleasure of hosting (jointly with Martin Grötschel), namely the 50-th International Mathematical Olympiad, held in Bremen in 2009, at which several premier IMO gold medal winners got on stage to talk about the mathematics that they studied, the mathematics that they are studying, and the mathematics that they are interested in.

All this is reflected in this volume, which contains some of these IMO presentations, as well as other facets of the mathematics research experience. It was put together with admirable care, energy, and attention to detail by Dierk Schleicher (one of the chief organizers of the 50-th IMO in Bremen) and Malte Lackmann (a successful three-time IMO participant). Let me express my gratitude to both of them for this volume, which I see as a book-length exposition of an answer to the question “What is Mathematics?” — and let me wish you an informative, enjoyable, and (in the proper sense of the word) *attractive* reading experience.

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