

Preface

Geometry is a pervasive mathematical concept that appears in many places and in many disguises. The representation of geometric entities, of their unions and intersections, and of their transformations is something that we, as human beings, can relate to particularly well, since these concepts are omnipresent in our everyday life. Being able to describe something in terms of geometric concepts lifts a purely formal description to something we can relate to. The application of “common geometrical sense” can therefore be extremely helpful in understanding many mathematical concepts. This is true not only for elements directly related to geometry such as the fundamental matrix, but also for polynomial curves such as Pythagorean hodograph curves, and analysis-related aspects such as the Cauchy–Riemann equations [127], to mention a few examples.

An algebra for geometry is therefore a desirable mathematical tool, and indeed many such algebras have been developed. The first ones were probably Grassmann’s algebra of extensive entities, Hamilton’s quaternions, and complex numbers, which were all combined by W. K. Clifford into a single algebraic system, *Clifford algebra*. The aim of this text is not only to show that Clifford algebra is *the* geometric algebra, but also to demonstrate its advantages for important engineering applications.

This text is split into two main parts: a theoretical and an application part. In the theoretical part, the mathematical foundations are laid and a methodology is presented for the representation and numerical implementation of geometric constraints, with possibly uncertain geometric entities. The application part demonstrates with a number of examples how this methodology can be applied, but also demonstrates that the representative power of geometric algebra can lead to interesting new results.

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