1 Multi-Objective Control of Time-Discrete Systems and Dynamic Games on Networks

1.1 Problem Formulation
   1.1.1 Single-Objective Discrete Control Problem
   1.1.2 Multi-Objective Control Based on the Concept of Non-cooperative Games: Nash Equilibria
   1.1.3 Hierarchical Control and Stackelberg’s Optimization Principle
   1.1.4 Multi-Objective Control Based on the Concept of Cooperative Games: Pareto Optima
   1.1.5 Stationary and Non-Stationary Control of Time-Discrete Systems

1.2 Multi-Objective Control of Time-Discrete Systems with Infinite Time Horizon

1.3 Alternate Players’ Control Condition and Nash Equilibria for Dynamic Games in Positional Form

1.4 Algorithms for Solving Single-Objective Control Problems on Networks
   1.4.1 Dynamic Programming Algorithms for Solving Optimal Control Problems on Networks
   1.4.2 An Extension of Dijkstra’s Algorithm for Optimal Control Problems with a Free Number of Stages

1.5 Multi-Objective Control and Non-Cooperative Games on Dynamic Networks
   1.5.1 The Problem of Determining the Optimal Stationary Strategies in a Dynamic c-Game
   1.5.2 The Problem of Determining the Optimal Non-Stationary Strategies in a Dynamic c-Game

1.6 Main Results for Dynamic c-Games with Constant Costs of the Edges and Determining Optimal Stationary Strategies of the Players
1.7 Computational Complexity of the Problem of Determining
Optimal Stationary Strategies in a Dynamic c-Game ........ 45
1.8 Determining the Optimal Stationary Strategies for a Dynamic
c-Game with Non-Constant Cost Functions on the Edges ...... 45
1.9 Determining Nash Equilibria for Non-Stationary Dynamic
c-Games ................................................ 53
  1.9.1 Time-Expanded Networks for Non-Stationary
       Dynamic c-Games and Their Main Properties .......... 53
  1.9.2 Determining Nash Equilibria ....................... 55
1.10 Application of the Dynamic c-Game for Studying and Solving
    Multi-Objective Control Problems .......................... 57
1.11 Multi-Objective Control and Cooperative Games on Dynamic
    Networks ................................................. 58
  1.11.1 Stationary Strategies on Networks and Pareto Solutions 58
  1.11.2 A Pareto Solution for the Problem with Non-
       Stationary Strategies on Networks ...................... 59
1.12 Determining Pareto Solutions for Multi-Objective Control
    Problems on Networks ................................. 60
  1.12.1 Determining Pareto Stationary Strategies .......... 60
  1.12.2 Pareto Solution for the Non-Stationary Case of the
       Problem .............................................. 65
  1.12.3 Computational Complexity of the Stationary Case
       of the Problem and an Algorithm for its Solving on
       Acyclic Networks ..................................... 65
1.13 Determining Pareto Optima for Multi-Objective Control
    Problems ............................................... 66
1.14 Determining a Stackelberg Solution for Hierarchical Control
    Problems ............................................... 67
  1.14.1 A Stackelberg Solution for Static Games .......... 68
  1.14.2 Hierarchical Control on Networks and Determining
       Stackelberg Stationary Strategies ...................... 69
  1.14.3 An Algorithm for Determining Stackelberg Stationary
       Strategies on Acyclic Networks ......................... 73
  1.14.4 An Algorithm for Solving Hierarchical Control Problems 78

2 Max-Min Control Problems and Solving Zero-Sum Games
   on Networks .............................................. 81
  2.1 Discrete Control and Finite Antagonistic Dynamic Games ... 81
  2.2 Max-Min Control Problem with Infinite Time Horizon ...... 82
  2.3 Zero-Sum Games on Networks and a Polynomial Time
       Algorithm for Max-Min Paths Problems .................. 83
    2.3.1 Problem Formulation ............................. 84
    2.3.2 An Algorithm for Solving the Problem on Acyclic
           Networks ........................................... 86
    2.3.3 Main Results for the Problem on an Arbitrary Network 88
2.3.4 A Polynomial Time Algorithm for Determining Optimal Strategies of the Players in a Dynamic $c$-Game 90

2.3.5 A Pseudo-Polynomial Time Algorithm for Solving a Dynamic $c$-Game .............................. 95

2.4 A Polynomial Time Algorithm for Solving Acyclic $l$-Games on Networks ............................................ 101

2.4.1 Problem Formulation ........................................ 101

2.4.2 Main Properties of Optimal Strategies in Acyclic $l$-Games ......................................................... 102

2.4.3 A Polynomial Time Algorithm for Finding the Value and the Optimal Strategies in an Acyclic $l$-Game ...... 103

2.5 Cyclic Games: Algorithms for Finding the Value and the Optimal Strategies of the Players .......................... 105

2.5.1 Problem Formulation and Main Properties ................. 106

2.5.2 Determining the Best Response of the First Player for a Fixed Strategy of the Second Player .................. 107

2.5.3 Some Preliminary Results ................................... 110

2.5.4 The Reduction of Cyclic Games to Ergodic Games ...... 111

2.5.5 A Polynomial Time Algorithm for Solving Ergodic Zero-Value Cyclic Games ................................. 111

2.5.6 A Polynomial Time Algorithm for Solving Cyclic Games Based on the Reduction to Acyclic $l$-Games ....... 113

2.5.7 An Approach for Solving Cyclic Games Based on a Dichotomy Method and Solving Dynamic $c$-Games .... 116

2.6 Cyclic Games with Random States’ Transitions of the Dynamical System ........................................ 117

2.7 A Nash Equilibria Condition for Cyclic Games with $p$ Players ......................................................... 118

2.8 Determining Pareto Optima for Cyclic Games with $p$ Players ......................................................... 122

3 Extension and Generalization of Discrete Control Problems and Algorithmic Approaches for its Solving .... 125

3.1 Discrete Control Problems with Varying Time of States’ Transitions of the Dynamical System ................. 125

3.1.1 The Single-Objective Control Problem with Varying Time of States’ Transitions of the Dynamical System .. 126

3.1.2 An Algorithm for Solving a Single-Objective Control Problem with Varying Time of States’ Transitions of the Dynamical System ......................................................... 127

3.1.3 The Discrete Control Problem with Cost Functions of System’s Passages that Depend on the Transition-Time of States’ Transitions ......................................................... 132

3.2 The Control Problem on a Network with Transition-Time Functions on the Edges ................................. 133

3.2.1 Problem Formulation ........................................ 133
3.2.2 An Algorithm for Solving the Problem on a Network with Transition-Time Functions on the Edges..........134
3.3 Multi-Objective Control of Time-Discrete Systems with Varying Time of States’ Transitions.................141
  3.3.1 Multi-Objective Discrete Control with Varying Time of States’ Transitions of Dynamical Systems.........141
  3.3.2 A Dynamic c-Game on Networks with Transition-Time Functions on the Edges.........................146
  3.3.3 Remark on Determining Pareto Optima for the Multi-Objective Control Problem with Varying Time of States’ Transitions........................................149
3.4 An Algorithm for Solving the Discrete Optimal Control Problem with Infinite Time Horizon and Varying Time of the States’ Transitions........................................150
  3.4.1 Problem Formulation and Some Preliminary Results.........150
  3.4.2 An Algorithm for Determining an Optimal Stationary Control for Dynamical Systems with Infinite Time Horizon.........................................................152
3.5 A General Approach for Algorithmic Solutions of Discrete Optimal Control Problems and its Game-Theoretic Extension.154
  3.5.1 A General Optimal Control Model..........................154
  3.5.2 An Algorithm for Determining an Optimal Solution of the Problem with Fixed Starting and Final States....156
  3.5.3 The Discrete Optimal Control Problem on a Network...159
  3.5.4 The Game-Theoretic Control Model with p Players........160
  3.5.5 The Game-Theoretic Control Problem on Networks and an Algorithm for its Solving.....................161
  3.5.6 Multi-Criteria Discrete Control Problems: Pareto Optima........................................169
3.6 Pareto-Nash Equilibria for Multi-Objective Games.............171
  3.6.1 Problem Formulation..........................172
  3.6.2 Main Results..................................173
  3.6.3 Discrete and Matrix Multi-Objective Games.............177
  3.6.4 Some Comments on and Interpretations of Multi-Objective Games........................................179
  3.6.5 Determining a Pareto-Stackelberg Solution for Multi-Objective Games.....................................179
4 Discrete Control and Optimal Dynamic Flow Problems on Networks........................................181
  4.1 Single-Commodity Dynamic Flow Problems and the Time-Expanded Network Method for Their Solving........181
    4.1.1 The Minimum Cost Dynamic Flow Problem........182
    4.1.2 The Main Results................................183
    4.1.3 The Dynamic Model with Flow Storage at Nodes....186
4.1.4 The Dynamic Model with Flow Storage at Nodes and
Integral Constant Demand-Supply Functions ............ 188
4.1.5 The Algorithm ........................................ 189
4.1.6 Constructing the Time-Expanded Network and its Size . 190
4.1.7 Approaches for Solving the Minimum Cost Flow
Problem with Different Types of Cost Functions on
the Edges .................................................. 200
4.1.8 Determining the Minimum Cost Flows in Dynamic
Networks with Transition Time Functions that
Depend on Flow and Time .................................... 208
4.1.9 An Algorithm for Solving the Maximum Dynamic
Flow Problem .................................................. 212
4.2 Multi-Commodity Dynamic Flow Problems and Algorithms
for their Solving ............................................ 214
4.2.1 The Minimum Cost Multi-Commodity Dynamic Flow
Problem .................................................. 214
4.2.2 The Main Results ........................................ 216
4.2.3 The Algorithm .......................................... 220
4.2.4 Examples .............................................. 220
4.2.5 The Dynamic Multi-Commodity Minimum Cost Flow
Problem with Transition Time Functions that Depend
on Flows and on Time ............................................. 224
4.2.6 Generalizations ........................................ 229
4.2.7 An Algorithm for Solving the Maximum Dynamic
Multi-Commodity Flow Problem ............................. 229
4.3 The Game-Theoretic Approach for Dynamic Flow Problems
on Networks .................................................... 231

5 Applications and Related Topics .............................. 233
5.1 Analysis and Control of Time-Discrete Systems:
Resource Planning - The TEM Model ........................ 233
5.1.1 Motivation .............................................. 234
5.1.2 The Basic Model ...................................... 234
5.1.3 Control Theoretic Part .................................. 237
5.1.4 Problem of Fixed Point Controllability and
Null-Controllability ......................................... 238
5.1.5 Optimal Investment Parameter ........................ 240
5.1.6 A Game-Theoretic Extension -
Relation to Multilayered Decision Problems ............ 244
5.2 Algorithmic Solutions for an Emission Reduction Game: The
Kyoto Game .............................................. 250
5.2.1 The Core in the TEM Model ............................. 250
5.2.2 A Second Cooperative Treatment of the TEM Model . 259
5.2.3 Comments ............................................... 268
Optimization and Multiobjective Control of Time–Discrete Systems
Dynamic Networks and Multilayered Structures
Lozovanu, D.
2009, XVI, 285 p. 53 illus., Hardcover
ISBN: 978-3-540-85024-3