Contents

CHAPTER I
Nilpotent Lie Algebras and Solvable Lie Algebras 1
  1. Lower Central Series 1
  2. Definition of Nilpotent Lie Algebras 1
  3. An Example of a Nilpotent Algebra 2
  4. Engel's Theorems 2
  5. Derived Series 3
  6. Definition of Solvable Lie Algebras 3
  7. Lie's Theorem 4
  8. Cartan's Criterion 4

CHAPTER II
Semisimple Lie Algebras (General Theorems) 5
  1. Radical and Semisimplicity 5
  2. The Cartan–Killing Criterion 6
  3. Decomposition of Semisimple Lie Algebras 6
  4. Derivations of Semisimple Lie Algebras 7
  5. Semisimple Elements and Nilpotent Elements 7
  6. Complete Reducibility Theorem 8
  7. Complex Simple Lie Algebras 8
  8. The Passage from Real to Complex 9

CHAPTER III
Cartan Subalgebras 10
  1. Definition of Cartan Subalgebras 10
  2. Regular Elements: Rank 10
3. The Cartan Subalgebra Associated with a Regular Element 11
4. Conjugacy of Cartan Subalgebras 12
5. The Semisimple Case 15
6. Real Lie Algebras 16

CHAPTER IV
The Algebra $\mathfrak{sl}_2$ and Its Representations 17
1. The Lie Algebra $\mathfrak{sl}_2$ 17
2. Modules, Weights, Primitive Elements 17
3. Structure of the Submodule Generated by a Primitive Element 18
4. The Modules $W_n$ 19
5. Structure of the Finite-Dimensional $\mathfrak{g}$-Modules 20
6. Topological Properties of the Group $SL_2$ 21
7. Applications 22

CHAPTER V
Root Systems 24
1. Symmetries 24
2. Definition of Root Systems 25
3. First Examples 26
4. The Weyl Group 27
5. Invariant Quadratic Forms 27
6. Inverse Systems 28
7. Relative Position of Two Roots 29
8. Bases 30
9. Some Properties of Bases 31
10. Relations with the Weyl Group 33
11. The Cartan Matrix 34
12. The Coxeter Graph 35
13. Irreducible Root Systems 36
14. Classification of Connected Coxeter Graphs 37
15. Dynkin Diagrams 38
17. Complex Root Systems 41

CHAPTER VI
Structure of Semisimple Lie Algebras 43
1. Decomposition of $\mathfrak{g}$ 43
2. Proof of Theorem 2 45
3. Borel Subalgebras 47
4. Weyl Bases 48
5. Existence and Uniqueness Theorems 50
6. Chevalley's Normalization 51
Appendix. Construction of Semisimple Lie Algebras by Generators and Relations 52
CHAPTER VII
Linear Representations of Semisimple Lie Algebras 56

1. Weights 56
2. Primitive Elements 57
3. Irreducible Modules with a Highest Weight 58
4. Finite-Dimensional Modules 60
5. An Application to the Weyl Group 62
6. Example: sl_{n+1} 62
7. Characters 63
8. H. Weyl's formula 64

CHAPTER VIII
Complex Groups and Compact Groups 66

1. Cartan Subgroups 66
2. Characters 67
3. Relations with Representations 68
4. Borel Subgroups 68
5. Construction of Irreducible Representations from Borel Subgroups 69
6. Relations with Algebraic Groups 70
7. Relations with Compact Groups 70

Bibliography 72

Index 73
Complex Semisimple Lie Algebras
Serre, J.-P.
2001, IX, 74 p., Hardcover
ISBN: 978-3-540-67827-4