Contents

Volume I Basic Theory

1 Generalized Differentiation in Banach Spaces .......................... 3
  1.1 Generalized Normals to Nonconvex Sets ............................ 4
    1.1.1 Basic Definitions and Some Properties ........................ 4
    1.1.2 Tangential Approximations .................................... 12
    1.1.3 Calculus of Generalized Normals .............................. 18
    1.1.4 Sequential Normal Compactness of Sets ....................... 27
    1.1.5 Variational Descriptions and Minimality ................... 33
  1.2 Coderivatives of Set-Valued Mappings ................................ 39
    1.2.1 Basic Definitions and Representations ........................ 40
    1.2.2 Lipschitzian Properties ....................................... 47
    1.2.3 Metric Regularity and Covering ................................ 56
    1.2.4 Calculus of Coderivatives in Banach Spaces ................. 70
    1.2.5 Sequential Normal Compactness of Mappings ................ 75
  1.3 Subdifferentials of Nonsmooth Functions ............................ 81
    1.3.1 Basic Definitions and Relationships .......................... 82
    1.3.2 Fréchet-Like ε-Subgradients and Limiting Representations .... 87
    1.3.3 Subdifferentiation of Distance Functions .................... 97
    1.3.4 Subdifferential Calculus in Banach Spaces ................. 112
    1.3.5 Second-Order Subdifferentials ............................... 121
  1.4 Commentary to Chap. 1 ........................................... 132

2 Extremal Principle in Variational Analysis .............................. 171
  2.1 Set Extremality and Nonconvex Separation .......................... 172
    2.1.1 Extremal Systems of Sets .................................... 172
    2.1.2 Versions of the Extremal Principle and Supporting Properties ... 174
    2.1.3 Extremal Principle in Finite Dimensions .................... 178
  2.2 Extremal Principle in Asplund Spaces ............................... 180
2.2.1 Approximate Extremal Principle in Smooth Banach Spaces .......................... 180
2.2.2 Separable Reduction ............................................. 183
2.2.3 Extremal Characterizations of Asplund Spaces .......... 195

2.3 Relations with Variational Principles .......................... 203
2.3.1 Ekeland Variational Principle ........................................... 204
2.3.2 Subdifferential Variational Principles .................. 206
2.3.3 Smooth Variational Principles ................................. 210

2.4 Representations and Characterizations in Asplund Spaces ........ 214
2.4.1 Subgradients, Normals, and Coderivatives in Asplund Spaces ............................................... 214
2.4.2 Representations of Singular Subgradients and Horizontal Normals to Graphs and Epigraphs ...... 223

2.5 Versions of Extremal Principle in Banach Spaces ........... 230
2.5.1 Axiomatic Normal and Subdifferential Structures .......... 231
2.5.2 Specific Normal and Subdifferential Structures ......... 235
2.5.3 Abstract Versions of Extremal Principle ............... 245

2.6 Commentary to Chap. 2 .................................. 249

3 Full Calculus in Asplund Spaces ............................ 261
3.1 Calculus Rules for Normals and Coderivatives .................. 261
3.1.1 Calculus of Normal Cones ............................................. 262
3.1.2 Calculus of Coderivatives ........................................... 274
3.1.3 Strictly Lipschitzian Behavior and Coderivative Scalarization ........................................... 287

3.2 Subdifferential Calculus and Related Topics ................. 296
3.2.1 Calculus Rules for Basic and Singular Subgradients ...... 296
3.2.2 Approximate Mean Value Theorem with Some Applications .................. 308
3.2.3 Connections with Other Subdifferentials .................. 317
3.2.4 Graphical Regularity of Lipschitzian Mappings .......... 327
3.2.5 Second-Order Subdifferential Calculus .................. 335

3.3 SNC Calculus for Sets and Mappings ......................... 341
3.3.1 Sequential Normal Compactness of Set Intersections and Inverse Images ........................................... 341
3.3.2 Sequential Normal Compactness for Sums and Related Operations with Maps .................. 349
3.3.3 Sequential Normal Compactness for Compositions of Maps ........................................... 354

3.4 Commentary to Chap. 3 .................................. 361

4 Characterizations of Well-Posedness and Sensitivity Analysis .......................... 377
4.1 Neighborhood Criteria and Exact Bounds ................... 378
4.1.1 Neighborhood Characterizations of Covering .................. 378
4.1.2 Neighborhood Characterizations of Metric Regularity and Lipschitzian Behavior ............................................. 382
4.2 Pointbased Characterizations ............................................. 384
  4.2.1 Lipschitzian Properties via Normal and Mixed Coderivatives ................................................................. 385
  4.2.2 Pointbased Characterizations of Covering and Metric Regularity ............................................................... 394
  4.2.3 Metric Regularity under Perturbations ................................................. 399
4.3 Sensitivity Analysis for Constraint Systems ..................................... 406
  4.3.1 Coderivatives of Parametric Constraint Systems ........................................... 407
  4.3.2 Lipschitzian Stability of Constraint Systems ........................................... 414
4.4 Sensitivity Analysis for Variational Systems ..................................... 421
  4.4.1 Coderivatives of Parametric Variational Systems ........................................... 422
  4.4.2 Coderivative Analysis of Lipschitzian Stability ........................................... 436
  4.4.3 Lipschitzian Stability under Canonical Perturbations ........................................... 450
4.5 Commentary to Chap. 4 .......................................................... 462

Volume II Applications

5 Constrained Optimization and Equilibria ............................................. 3
  5.1 Necessary Conditions in Mathematical Programming ....................... 3
    5.1.1 Minimization Problems with Geometric Constraints ................. 4
    5.1.2 Necessary Conditions under Operator Constraints ..................... 9
    5.1.3 Necessary Conditions under Functional Constraints ............... 22
    5.1.4 Suboptimality Conditions for Constrained Problems ............... 41
  5.2 Mathematical Programs with Equilibrium Constraints .................... 46
    5.2.1 Necessary Conditions for Abstract MPECs ......................... 47
    5.2.2 Variational Systems as Equilibrium Constraints ..................... 51
    5.2.3 Refined Lower Subdifferential Conditions for MPECs via Exact Penalization ............................................. 61
  5.3 Multiobjective Optimization .................................................. 69
    5.3.1 Optimal Solutions to Multiobjective Problems ....................... 70
    5.3.2 Generalized Order Optimality ............................................ 73
    5.3.3 Extremal Principle for Set-Valued Mappings .......................... 83
    5.3.4 Optimality Conditions with Respect to Closed Preferences .......... 92
    5.3.5 Multiobjective Optimization with Equilibrium Constraints .......... 99
  5.4 Subextremality and Suboptimality at Linear Rate .......................... 109
    5.4.1 Linear Subextremality of Set Systems .................................. 110
    5.4.2 Linear Suboptimality in Multiobjective Optimization ............. 115
    5.4.3 Linear Suboptimality for Minimization Problems ................... 125
  5.5 Commentary to Chap. 5 .......................................................... 131
6  Optimal Control of Evolution Systems in Banach Spaces . . 159
6.1 Optimal Control of Discrete-Time and Continuous-time Evolution Inclusions ........................................... 160
  6.1.1 Differential Inclusions and Their Discrete Approximations ......................................................... 160
  6.1.2 Bolza Problem for Differential Inclusions and Relaxation Stability ................................................. 168
  6.1.3 Well-Posed Discrete Approximations of the Bolza Problem ..................................................... 175
  6.1.4 Necessary Optimality Conditions for Discrete-Time Inclusions ................................................. 184
  6.1.5 Euler-Lagrange Conditions for Relaxed Minimizers ........................................................................ 198
6.2 Necessary Optimality Conditions for Differential Inclusions without Relaxation ........................................... 210
  6.2.1 Euler-Lagrange and Maximum Conditions for Intermediate Local Minimizers .......................... 211
  6.2.2 Discussion and Examples ................................................................................................................. 219
6.3 Maximum Principle for Continuous-Time Systems with Smooth Dynamics .............................................. 227
  6.3.1 Formulation and Discussion of Main Results ................................................................................. 228
  6.3.2 Maximum Principle for Free-Endpoint Problems ............................................................................ 234
  6.3.3 Transversality Conditions for Problems with Inequality Constraints ......................................... 239
  6.3.4 Transversality Conditions for Problems with Equality Constraints ........................................... 244
6.4 Approximate Maximum Principle in Optimal Control .............................................................................. 248
  6.4.1 Exact and Approximate Maximum Principles for Discrete-Time Control Systems ....................... 248
  6.4.2 Uniformly Upper Subdifferentiable Functions .............................................................................. 254
  6.4.3 Approximate Maximum Principle for Free-Endpoint Control Systems ........................................ 258
  6.4.4 Approximate Maximum Principle under Endpoint Constraints: Positive and Negative Statements 268
  6.4.5 Approximate Maximum Principle under Endpoint Constraints: Proofs and Applications .......... 276
6.5 Commentary to Chap. 6 ......................................................................................................................... 297
7  Optimal Control of Distributed Systems ................................................................................................. 335
7.1 Optimization of Differential-Algebraic Inclusions with Delays ............................................................. 336
  7.1.1 Discrete Approximations of Differential-Algebraic Inclusions .................................................... 338
  7.1.2 Strong Convergence of Discrete Approximations ......................................................................... 346
7.1.3 Necessary Optimality Conditions  
for Difference-Algebraic Systems .................... 352
7.1.4 Euler-Lagrange and Hamiltonian Conditions  
for Differential-Algebraic Systems .................. 357

7.2 Neumann Boundary Control  
of Semilinear Constrained Hyperbolic Equations ........... 364
7.2.1 Problem Formulation and Necessary Optimality  
Conditions for Neumann Boundary Controls ............ 365
7.2.2 Analysis of State and Adjoint Systems  
in the Neumann Problem ........................... 369
7.2.3 Needle-Type Variations and Increment Formula ...... 376
7.2.4 Proof of Necessary Optimality Conditions ............ 380

7.3 Dirichlet Boundary Control  
of Linear Constrained Hyperbolic Equations ................ 386
7.3.1 Problem Formulation and Main Results  
for Dirichlet Controls .............................. 387
7.3.2 Existence of Dirichlet Optimal Controls ............. 390
7.3.3 Adjoint System in the Dirichlet Problem ............. 391
7.3.4 Proof of Optimality Conditions ..................... 395

7.4 Minimax Control of Parabolic Systems  
with Pointwise State Constraints ........................ 398
7.4.1 Problem Formulation and Splitting .................. 400
7.4.2 Properties of Mild Solutions  
and Minimax Existence Theorem .................... 404
7.4.3 Suboptimality Conditions for Worst Perturbations .... 410
7.4.4 Suboptimal Controls under Worst Perturbations ...... 422
7.4.5 Necessary Optimality Conditions  
under State Constraints ........................... 427

7.5 Commentary to Chap. 7 .................................. 439

8 Applications to Economics .................................. 461
8.1 Models of Welfare Economics .......................... 461
8.1.1 Basic Concepts and Model Description ............... 462
8.1.2 Net Demand Qualification Conditions for Pareto  
and Weak Pareto Optimal Allocations .................. 465
8.2 Second Welfare Theorem for Nonconvex Economies .......... 468
8.2.1 Approximate Versions of Second Welfare Theorem .... 469
8.2.2 Exact Versions of Second Welfare Theorem .......... 474
8.3 Nonconvex Economies with Ordered Commodity Spaces ...... 477
8.3.1 Positive Marginal Prices ........................ 477
8.3.2 Enhanced Results for Strong Pareto Optimality ...... 479
8.4 Abstract Versions and Further Extensions ................ 484
8.4.1 Abstract Versions of Second Welfare Theorem ....... 484
8.4.2 Public Goods and Restriction on Exchange .......... 490
8.5 Commentary to Chap. 8 ............................... 492
XXII  Contents

References .......................................................... 477
List of Statements .................................................... 543
Glossary of Notation ............................................. 565
Subject Index ......................................................... 569
Variational Analysis and Generalized Differentiation I
Basic Theory
Mordukhovich, B.S.
2006, XXII, 579 p., Hardcover
ISBN: 978-3-540-25437-9