Contents

Part I A Guided Tour to Arbitrage Theory

1 The Story in a Nutshell .............................................. 3
  1.1 Arbitrage ............................................................. 3
  1.2 An Easy Model of a Financial Market ............................. 4
  1.3 Pricing by No-Arbitrage ........................................... 5
  1.4 Variations of the Example ........................................ 7
  1.5 Martingale Measures .............................................. 7
  1.6 The Fundamental Theorem of Asset Pricing ...................... 8

2 Models of Financial Markets on Finite Probability Spaces 11
  2.1 Description of the Model ......................................... 11
  2.2 No-Arbitrage and the Fundamental Theorem of Asset Pricing 16
  2.3 Equivalence of Single-period with Multiperiod Arbitrage ....... 22
  2.4 Pricing by No-Arbitrage ........................................... 23
  2.5 Change of Numéraire ............................................... 27
  2.6 Kramkov’s Optional Decomposition Theorem ..................... 31

3 Utility Maximisation on Finite Probability Spaces ............ 33
  3.1 The Complete Case ............................................... 34
  3.2 The Incomplete Case ............................................... 41
  3.3 The Binomial and the Trinomial Model ........................... 45

4 Bachelier and Black-Scholes ........................................ 57
  4.1 Introduction to Continuous Time Models ........................ 57
  4.2 Models in Continuous Time ...................................... 57
  4.3 Bachelier’s Model .................................................. 58
  4.4 The Black-Scholes Model ......................................... 60
## Contents

### 5 The Kreps-Yan Theorem

- 5.1 A General Framework .................................................. 71
- 5.2 No Free Lunch .............................................................. 76

### 6 The Dalang-Morton-Willinger Theorem

- 6.1 Statement of the Theorem ............................................. 85
- 6.2 The Predictable Range .................................................. 86
- 6.3 The Selection Principle .................................................. 89
- 6.4 The Closedness of the Cone $C$ ........................................ 92
- 6.5 Proof of the Dalang-Morton-Willinger Theorem for $T = 1$ .... 94
- 6.6 A Utility-based Proof of the DMW Theorem for $T = 1$ ......... 96
- 6.7 Proof of the Dalang-Morton-Willinger Theorem for $T \geq 1$
  by Induction on $T$ ...................................................... 102
- 6.8 Proof of the Closedness of $K$ in the Case $T \geq 1$ ............. 103
- 6.9 Proof of the Closedness of $C$ in the Case $T \geq 1$
  under the (NA) Condition .............................................. 105
- 6.10 Proof of the Dalang-Morton-Willinger Theorem for $T \geq 1$
  using the Closedness of $C$ ............................................. 107
- 6.11 Interpretation of the $L^\infty$-Bound in the DMW Theorem ...... 108

### 7 A Primer in Stochastic Integration

- 7.1 The Set-up ................................................................. 111
- 7.2 Introduction on Stochastic Processes ................................. 112
- 7.3 Strategies, Semi-martingales and Stochastic Integration ........ 117

### 8 Arbitrage Theory in Continuous Time: an Overview

- 8.1 Notation and Preliminaries ............................................ 129
- 8.2 The Crucial Lemma ..................................................... 131
- 8.3 Sigma-martingales and the Non-locally Bounded Case ............. 140

### Part II The Original Papers

### 9 A General Version of the Fundamental Theorem
  of Asset Pricing (1994)

- 9.1 Introduction ............................................................... 149
- 9.2 Definitions and Preliminary Results .................................. 155
- 9.3 No Free Lunch with Vanishing Risk .................................. 160
- 9.4 Proof of the Main Theorem ............................................ 164
- 9.5 The Set of Representing Measures .................................... 181
- 9.6 No Free Lunch with Bounded Risk .................................... 186
- 9.7 Simple Integrands ....................................................... 190
- 9.8 Appendix: Some Measure Theoretical Lemmas ....................... 202
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>Introduction and Known Results</td>
<td>207</td>
</tr>
<tr>
<td>10.2</td>
<td>Construction of the Example</td>
<td>210</td>
</tr>
<tr>
<td>10.3</td>
<td>Incomplete Markets</td>
<td>212</td>
</tr>
<tr>
<td>11.1</td>
<td>Introduction</td>
<td>217</td>
</tr>
<tr>
<td>11.2</td>
<td>Basic Theorems</td>
<td>219</td>
</tr>
<tr>
<td>11.3</td>
<td>Duality Relation</td>
<td>222</td>
</tr>
<tr>
<td>11.4</td>
<td>Hedging and Change of Numéraire</td>
<td>225</td>
</tr>
<tr>
<td>12</td>
<td>The Existence of Absolutely Continuous Local Martingale Measures (1995)</td>
<td>231</td>
</tr>
<tr>
<td>12.1</td>
<td>Introduction</td>
<td>231</td>
</tr>
<tr>
<td>12.2</td>
<td>The Predictable Radon-Nikodým Derivative</td>
<td>235</td>
</tr>
<tr>
<td>12.3</td>
<td>The No-Arbitrage Property and Immediate Arbitrage</td>
<td>239</td>
</tr>
<tr>
<td>12.4</td>
<td>The Existence of an Absolutely Continuous Local Martingale Measure</td>
<td>244</td>
</tr>
<tr>
<td>13</td>
<td>The Banach Space of Workable Contingent Claims in Arbitrage Theory (1997)</td>
<td>251</td>
</tr>
<tr>
<td>13.1</td>
<td>Introduction</td>
<td>251</td>
</tr>
<tr>
<td>13.2</td>
<td>Maximal Admissible Contingent Claims</td>
<td>255</td>
</tr>
<tr>
<td>13.3</td>
<td>The Banach Space Generated by Maximal Contingent Claims</td>
<td>261</td>
</tr>
<tr>
<td>13.4</td>
<td>Some Results on the Topology of $\mathcal{G}$</td>
<td>266</td>
</tr>
<tr>
<td>13.5</td>
<td>The Value of Maximal Admissible Contingent Claims on the Set $\mathcal{M}^e$</td>
<td>272</td>
</tr>
<tr>
<td>13.6</td>
<td>The Space $\mathcal{G}$ under a Numéraire Change</td>
<td>274</td>
</tr>
<tr>
<td>13.7</td>
<td>The Closure of $\mathcal{G}^\infty$ and Related Problems</td>
<td>276</td>
</tr>
<tr>
<td>14.1</td>
<td>Introduction</td>
<td>279</td>
</tr>
<tr>
<td>14.2</td>
<td>Sigma-martingales</td>
<td>280</td>
</tr>
<tr>
<td>14.3</td>
<td>One-period Processes</td>
<td>284</td>
</tr>
<tr>
<td>14.4</td>
<td>The General $\mathbb{R}^d$-valued Case</td>
<td>294</td>
</tr>
<tr>
<td>14.5</td>
<td>Duality Results and Maximal Elements</td>
<td>305</td>
</tr>
<tr>
<td>15</td>
<td>A Compactness Principle for Bounded Sequences of Martingales with Applications (1999)</td>
<td>319</td>
</tr>
<tr>
<td>15.1</td>
<td>Introduction</td>
<td>319</td>
</tr>
<tr>
<td>15.2</td>
<td>Notations and Preliminaries</td>
<td>326</td>
</tr>
</tbody>
</table>
Contents

15.3 An Example ........................................... 332
15.4 A Substitute of Compactness
   for Bounded Subsets of $\mathcal{H}^1$ ......................... 334
   15.4.1 Proof of Theorem 15.A ............................... 335
   15.4.2 Proof of Theorem 15.C ............................... 337
   15.4.3 Proof of Theorem 15.B ............................... 339
   15.4.4 A proof of M. Yor’s Theorem ......................... 345
   15.4.5 Proof of Theorem 15.D ............................... 346
15.5 Application ........................................... 352

Part III Bibliography

References ..................................................... 359
The Mathematics of Arbitrage
Delbaen, F.; Schachermayer, W.
2006, XVI, 371 p., Hardcover
ISBN: 978-3-540-21992-7