Preface

There seems to be an ever-increasing need to get to the basic mathematics behind applications as quickly as possible. Optimization is one such body of ideas that permeates all of Science and Engineering. Quite often, interested readers who look at optimization ideas in a utilitarian way find that they need to dive into textbooks requiring a lot of formal training in order to find the basic concepts and techniques that they are seeking. This is quite frustrating at times, for all they require is a well-founded and educated intuition that will endow them with a basic understanding and some affordable computational tools to tackle problems in practice. In addition, even if a person is solely interested in a particular body of ideas in Optimization, he/she may need to know a little of everything so as to gain an overall mental picture. Real-life applications may demand identification of the nature of problems, the framework in which they can be treated, some simple computational methods to deal with equivalent toy models, etc. Once this initial fundamental information has been appropriately defined, further specialized literature may have to be sought. This text aims, as one way among various possibilities, to endow readers with such a basic overall background in Optimization that may enable them to identify a problem as a mathematical program, linear or nonlinear, or as a continuous optimization situation, in the form of either a variational problem or an optimal control case. It is also my experience that students who discover that it is virtually impossible to solve by hand even the more innocent-looking optimization problems become frustrated when they realize that in order to play with approximated solutions of simple variations of those situations, they will need to wait until much more has been learned about how to approximate optimal solutions. To propose one basic, systematic, and coherent solution for this approximation issue across all main areas of Optimization is also a main motivation for this text. It is just one affordable possibility among many others, with no intention of competing with more sophisticated and accurate methods in the computational arena. I describe some simple, yet non-entirely trivial, procedures to approximate optimal solutions of easy problems so that students may experience the joy of seeing with their own eyes optimal solutions of problems. There are, as just pointed out, other good and reasonable possibilities and solutions for this computational issue.
I feel this book is unique in that it makes an attempt to integrate all those main fields that share sufficient features to be placed under the umbrella of Optimization, a huge area of work in which many people are making progress on a daily basis. But there are so many distinct elements among those subareas that it is scarcely possible to make a deep analysis, within a single text, that covers all of them. In this sense, our treatment is basic and elementary but, as already pointed out in the preceding paragraph, it seeks to draw an overall picture of the interconnection among the main parts of Optimization that may be helpful to students.

The book will be of help to students of Science and Engineering, at both undergraduate and graduate level, who for the first time need to acquaint themselves with the basic principles of all areas of Optimization. In particular, the book ought to suffice for the content of typical courses in Science and Engineering majors. Familiarity with basic linear algebra, multivariate calculus, and basic differential equations is a fundamental prerequisite. Senior researchers whose main field of expertise is not Optimization, but who need to understand the basic theory and eventually to use approximation techniques for the problems in which they are interested, may also find these pages useful. I warn experts in some of the areas briefly covered in the text that they may find the coverage a little disappointing since the treatment of each area is too basic to be of interest for specialists. Some material or concepts that are regarded as fundamental in some subfields of Optimization may be missing from this book. Let me again stress that my intention has been to provide one basic, cross-cutting source relevant to all important areas in Optimization. This obviously means that it is not possible to attain great depth within individual chapters.

The text may serve several purposes:

- It can enable the user to avoid all issues about approximation techniques and to focus on basic analytical ideas in all chapters.
- It can enable the user to concentrate on mathematical programming problems, including issues about approximation, either by using the ideas in the text concerning simulation in practice or by utilizing other software packages.
- It may be used as the basis for a course focusing on continuous optimization problems, including approximation techniques.
- It can serve other specific purposes when time is scarce and there is a need to concentrate on narrower objectives.

It may be taken as the basic textbook for an Optimization course for any Science or Engineering major, and also for Master’s courses. The text has a modular structure in the sense that even separate individual sections may serve as short introductions to specific topics or techniques, depending on needs.

I have paid a lot of attention to exercises as I believe them to be a fundamental part of a text aiming to support basic learning in any field of mathematics. New ideas and motivations have been associated with exercises in an attempt to make them more inspiring and enlightening. I have divided the exercises into three distinct categories: those supporting main analytical concepts, those aiming to provide
basic training, and those of a more advanced nature. About 160 exercises with full solutions are gathered in the final chapter. Most of these exercises have been tried out in the Optimization course that the author and other colleagues have taught in the Math Dep of UCLM over the past 15 years. In this regard, particular thanks go to E. Aranda, J.C. Bellido and A. Donoso. I do not claim to have gathered all possible problems for such an introductory course. I could hardly do so, as there are hundreds of examples scattered throughout the literature. My goal, rather, has been to propose enough exercises to ensure that they enable the reader to achieve a reasonably mature grasp of the main concepts I have tried to convey. Whenever possible, I have made an attempt to inspire further investigation through some of them. I have given the source of problems whenever known, but I cannot guarantee to have done so in a systematic way; thus, readers may know other sources for some of the problems that have not been indicated in the text. Many of the problems can be found in other textbooks, so I do not claim the slightest originality here. Needless to say, anyone using this book as a source text can propose many other exercises to students. As a matter of fact, many other exercises can be found in the bibliography I have selected.

Concerning references, I have not tried to be exhaustive, and some interesting resources are probably missing. My aim has been to provide a limited number of well-chosen items so as to encourage further investigation or to identify where to look for deeper answers to more advanced questions. Each part has its own final bibliography section, though some are common to several parts.

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