Preface

The book is devoted to the study of the structure of approximate solutions of discrete-time optimal control problems and approximate solutions of dynamic discrete-time two-player zero-sum games. We present a number of results on properties of approximate solutions which are independent of the length of the interval, for all sufficiently large intervals. These results were obtained by the author during the last five years. Two types of results on the structure of solutions on large intervals are considered in the book. The results of the first type deal with the so-called turnpike property of optimal control problems, a term which was first coined by P. Samuelson in 1948 when he showed that an efficient expanding economy would spend most of the time in the vicinity of a balanced equilibrium path (also called a von Neumann path). To have the turnpike property means, roughly speaking, that the approximate solutions of the problems are determined mainly by the objective function (integrand) and are essentially independent of the choice of interval and endpoint conditions, except in regions close to the endpoints. Now, it is known that the turnpike property is a general phenomenon which holds for large classes of variational problems. For these classes of problems, using the Baire category (generic) approach, it was shown that the turnpike property holds for a generic (typical) variational problem. In this book, we are interested in individual turnpike results for discrete-time optimal control problems and for dynamic discrete-time two-player zero-sum games.

The results of the second type deal with the structure of approximate solutions of optimal control problems and zero-sum games in the regions close to the endpoints of the time intervals. It is shown that in these regions, solutions are close to solutions of the corresponding infinite horizon problems. This is a new direction in infinite horizon optimal control developed in the recent research of the author.

The monograph contains eight chapters. Chapter 1 is an introduction. In Chap. 2, we study the structure of approximate solutions of autonomous
nonconcave discrete-time optimal control Lagrange problems. In this chapter, our main goal is to analyze the structure of approximate solutions in regions close to the endpoints of the time intervals. In Chap. 3, we study the structure of approximate solutions of autonomous discrete-time optimal control problems which are discrete-time analogs of Bolza problems in the calculus of variations. They are described by a pair of objective functions which determines an optimality criterion. We consider two classes of Bolza problems and obtain for each of them the full description of approximate solutions of these problems on large intervals.

In Chap. 4, for discrete-time optimal control Bolza problems, it is shown that the turnpike phenomenon and the structure of solutions on finite intervals in the regions close to the endpoints are stable under small perturbations of the objective functions and the constraint sets.

In Chaps. 5–8, we analyze the structure of approximate solutions of two-player zero-sum games. In Chap. 5, we consider unconstrained dynamic games without using standard convexity–concavity assumptions and prove two turnpike results. Chapter 6 is devoted to the turnpike theory for a class of dynamic constrained games without using convexity–concavity assumptions. In Chap. 7, we study turnpike properties of approximate solutions for a class of nonautonomic dynamic discrete-time games with convexity–concavity assumptions. In Chap. 8, we study the existence and turnpike properties of approximate solutions for a class of dynamic constrained discrete-time two-player zero-sum games which satisfy convexity–concavity assumptions.

The results of Chaps. 2, 3, and 5–7 were obtained in our recent research, while the results of Chaps. 4 and 8 are new.

In the turnpike theory, there are many open problems, and it can be developed in many various directions. One of them is a generalization of the results presented in this book for problems with nonsingleton turnpikes. In principle, one can study turnpike properties of any variational or optimal control problem.
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