This book compiles the course notes on logic we have been taught to computer science students at the Universidade de Brasília during almost ten years. We decided to provide students the essential fundamentals on mathematical logic in an instrumental manner, restricting the discussion to only one relevant application of logic in computer science: logical deduction. Thus, the course notes provide the foundations of two different technologies to deal with logical deduction: natural deduction and Gentzen’s sequent calculus. Natural deduction is studied for the propositional and predicate calculi highlighting elements from this deductive system that discriminate between constructive and classical deduction and culminating with a presentation of Gödel’s completeness theorem. Gentzen’s sequent calculus is presented as an alternative technology that is proved to be equivalent to natural deduction. As for natural deduction, in this alternative deductive technology we highlight the elements that discriminate between constructive and classical deduction.

The instrumental part of these notes consists of the operationalization of the deductive rules of Gentzen’s sequent calculus in the context of proof assistants, using as computational framework the well-known Prototype Verification System (PVS). Connections between proof rules in this proof assistant and deductive rules in the sequent calculus are given and applications related with formal verification of properties of computational systems are illustrated through simple algebraic and algorithmic examples.

The principal motivation for the development of the notes is to offer undergraduate students of courses in engineering, computer science, and mathematics, the minimal theoretical background and most important, the minimal instrumental knowledge for the application of mathematical logic in the development of modern computer science. We found that this approach is adequate, since we detected that several students attending graduate courses on topics such as mathematical logic, type theory, proof theory and, in general on semantics of computation, despite being highly motivated, have a lack of the necessary basic knowledge and therefore are unable to apply elements of deductive logic that are used in nowadays computational artifacts. The essence of the problem is that they did not take logic
seriously since they did not realize that logic actually works as the cornerstone of several applications in computer science!

We are grateful to all the students, who have attended our courses and who have given us support as teaching assistants, and who have provided us valuable feedback, suggestions and corrections. In particular, we would like to thank Ariane Alves Almeida and Thiago Mendonça Ferreira Ramos for helping us in the PVS development of the sorting theory, which we have used to provide short course projects always related to the verification of simple algorithmic properties. This development is available in the web page that accompanies our notes: logic4CS.cic.unb.br. The authors are also grateful to Cesar Muñoz and other members of the Formal Methods group at NASA LaRC, as well as to Natarajan Shankar and Sam Owre from SRI International, the developers of PVS, for their kind support in issues related with the application and semantics of this proof assistant. This support was of great importance for the successful development of elaborated PVS theories by our research group at the Universidade de Brasília in which our students, now colleagues, André Luiz Galdino, Andréia Borges Avelar, Yuri Santos Rêgo and Ana Cristina Rocha-Oliveira played a paramount role. Despite all the received support, we would like to emphasize that all mistakes found in these course notes are our entire responsibility and, that we would be happy to receive all constructive feedbacks from the reader.

Last but not least, we would like to thank our families for understanding that academic work not only is done during working hours, but also requires hard work at home, and in particular to our wives, Mercedes and Tânia, to whom we dedicate this work.

Brasília D. F., Brazil
September 2016

Mauricio Ayala-Rincón
Flávio L.C. de Moura