The main goal of this book is to present background material and recently developed mathematical methods in the study of infinite-dimensional evolutionary models, taking into account dissipativity and stability properties of various forms and origins.

The main feature of dissipative systems is the presence of an energy reallocation mechanism with decaying in higher modes. This mechanism can lead to the appearance of complicated limit regimes and structures in the system, which are stable in a certain sense. It is commonly recognized that the general theory of dissipative systems was significantly stimulated in the 1980s with attempts to find adequate mathematical models to explain turbulence phenomena. By now, significant progress in the study of infinite-dimensional dissipative dynamics has been made (see, e.g., the monographs Babin/Vishik [9], Chepyzhov/Vishik [31], Chueshov [39], Hale [116], Ladyzhenskaya [142], Robinson [195], Sell/You [206], and Temam [216] and the references therein).

The main feature of this book in comparison with the sources mentioned is that we systematically present, develop, and use the quasi-stability method originally designed for second order in time models with nonlinear damping in collaboration with Irena Lasiecka in Chueshov/Lasiecka [56, 58] (see also our recent survey [60]). Here we extend this method substantially. New classes of second order evolutions, parabolic-type models, and PDE systems with delay are included for consideration.

We hope that this book will be useful not only to mathematicians interested in the general theory of dynamical systems, but also to physicists and engineers interested in both the mathematical background and methods for the asymptotic analysis of infinite-dimensional dissipative systems that arise in continuum mechanics.

Our presentation is based on general and abstract models and covers several important classes of nonlinear PDEs, which generate infinite-dimensional dissipative systems. These classes include heat and reaction-diffusion models, a wide spectrum of models arising in two-dimensional hydrodynamics for studying turbulence phenomena and plate and wave models with nonlinear state-dependent
damping. We also consider the nonlinearly damped wave Kirchhoff model and some classes of parabolic and hyperbolic delay problems.

Much of the analysis in this book is devoted to the stability of dynamics and a rigorous reduction of infinite-dimensional systems to some finite-dimensional structures, which are described only by finitely many degrees of freedom. These finite-dimensional structures should be of interest to application-oriented scientists, who pursue the mathematical simulation of real infinite-dimensional phenomena.

The book contains a large number of exercises. As in the famous monograph by Dan Henry [123], they are an integral part of the book. Most of them are placed strategically within the text, rather than at the end of a section. Some of the exercises are routine, while others are general comments and remarks written in “exercise form.” This allows us to make the narrative shorter and avoid extra refinement.

The book can be used as a textbook for courses in dissipative dynamics at the graduate level. It is sufficient to know the basic concepts and facts from functional analysis and ordinary differential equations to understand this book. In fact, many parts of the book were already used in advanced undergraduate and beginning graduate courses given by the author at the Kharkov University.

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