

# Chapter 2

## Turbulence in Rivers

Mário J. Franca and Maurizio Brocchini

**Abstract** The study of turbulence has always been a challenge for scientists working on geophysical flows. Turbulent flows are common in nature and have an important role in geophysical disciplines such as river morphology, landscape modeling, atmospheric dynamics and ocean currents. At present, new measurement and observation techniques suitable for fieldwork can be combined with laboratory and theoretical work to advance the understanding of river processes. Nevertheless, despite more than a century of attempts to correctly formalize turbulent flows, much still remains to be done by researchers and engineers working in hydraulics and fluid mechanics. In this contribution we introduce a general framework for the analysis of river turbulence. We revisit some findings and theoretical frameworks and provide a critical analysis of where the study of turbulence is important and how to include detailed information of this in the analysis of fluvial processes. We also provide a perspective of some general aspects that are essential for researchers/practitioners addressing the subject for the first time. Furthermore, we show some results of interest to scientists and engineers working on river flows.

**Keywords** Turbulence · Scales · Space-frame · Time-frame · River flow

### 2.1 Introduction

Similar to most flows of natural fluids, riverine flows are typically turbulent: turbulence is ubiquitous and represents a fundamental engine of transport, spreading and mixing. In particular, turbulence is the main sink of riverine flow total energy ( $E$ ). Large turbulent eddies are responsible for the conversion of total flow energy

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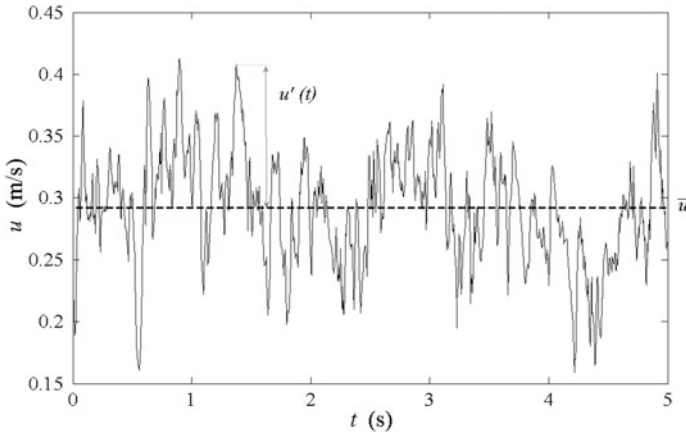
into turbulent energy; the former break down into mid-sized eddies, which in turn become increasingly smaller until they are destroyed by viscosity, wherein the flow energy is finally lost to heat. Furthermore, turbulence is responsible for physical processes in rivers such as the transport and mixing of diluted or undiluted substances, temperature transfer, sediment motion and suspension, and geomorphological evolution. Hence, the consideration of turbulence is fundamental to riverine environmental applications and requires the use of an adequate theoretical analysis [see, for instance, the amount of research dedicated to turbulence in fluvial hydraulics recently published in Schleiss et al. (2014)].

Although more than 120 years have passed since the first Reynolds experiments and after over 80 years since Taylor's first attempts to build a mathematical framework for turbulent flows, much still remains to be done and river turbulence is still a challenge for researchers and engineers working in hydraulics and fluid mechanics [paradigmatically, Enzo Levi names a section of his book *That Annoying Turbulence* (Levi 1995)]. However, a proper description of river turbulence is fundamental to the evolution of recently emerged ecologically oriented research areas pertaining to river flows, which include eco-geomorphology, bio-geomorphology, eco-hydrology, eco-hydraulics and environmental hydraulics (Nikora 2010).

The rather generic definition of a turbulent flow as a highly unpredictable flow characterized by many scales suggests some quantification for such a definition. Hence, when a variable of a fluid flow fluctuates in time or space, with non-zero second-order or higher statistical moments calculated along these dimensions, we are in the presence of a turbulent flow. Flow variables typically include velocity and pressure, although density, concentration of solids and temperature are also imprinted by the characteristics of turbulence. Typical of turbulence is the influence of many different interacting time/space scales in the dynamics of the flow.

The Reynolds decomposition, which was introduced more than 100 years ago (Levi 1995), divides fluid variables into a mean and a fluctuating field (Monin and Yaglom 1971; Frisch 1995; Pope 2000). This decomposition is applied to the hydrodynamic equations and it is the first step to account for the fluctuation of the variables due to turbulence (see Fig. 2.1 and, later, Sect. 2.2.2). The effect of turbulence in the flows involves the following processes, which may be properly formalized mathematically: ensemble advection by the mean flow; dispersion, which is due to the diversity of movements at different scales; turbulent advection or transport by eddies associated with the fluctuating field; and viscous or molecular diffusion due to agitation at very small scales (Chassaing 2000). These processes influence the environmental properties of the flows and represent the main engines of transport, mixing, entrainment, detrainment and dissipation.

Other structural characteristics associated with the time and space heterogeneity of the flows, which are thus associated with fluctuations in the fluid properties, include secondary currents and large-scale vortices either with horizontal or vertical axis, which may be considered *quasi*-steady structures, and waves (surface or internal) traveling within the river, with time scales that are large and with space scales that are comparable with the water depth and the river width. They may be conditioned or even generated by turbulent instability at smaller scales and, in turn,



**Fig. 2.1** Turbulent velocity signal and illustration of the application of Reynolds decomposition.  $t$  is the time in seconds,  $u$  is velocity, the *overbar* denotes turbulence-averaged components and  $'$  stands for the fluctuating component; see Sect. 2.2.2

may be responsible for changes in the overall turbulence of the flow, and include secondary currents in river bends (Blanckaert and De Vriend 2005; Termini and Piraino 2011; Nikora and Roy 2012; Abad et al. 2013), horizontal coherent vortices at the boundary between the main channel and floodplains (Knight and Shiono 1990; Shiono and Muto 1998; van Prooijen et al. 2005; Proust et al. 2013), and low-frequency large-scale flow structures traveling in low-submergence flows (Kirkbride and Ferguson 1995; Roy et al. 2004; Franca and Lemmin 2014).

The present paper is organized with a structure that focuses on the main methodological approaches used to address turbulent fluvial flows (e.g. time/space analysis, decomposition, etc.), rather than in terms of phenomenological aspects (e.g. coherent structures, mixing, etc.). The theoretical aspects discussed herein pertain to fluvial flows and concern the most common problems encountered by river engineers and scientists, namely, how to define time and spatial frameworks for turbulence analysis, the relation between turbulence and the transport of substances, diluted and undiluted (typical sediments), and simplifications that may be made in the analysis of river flows.

In the following, we first introduce basic definitions related to turbulence in rivers, including a discussion on scales characterizing fluvial flows; a presentation of the basic hydrodynamic equations used to address momentum, mass and species in the fluid; a description of energetic processes; and a discussion on several possible frameworks that can be used to study turbulent flows. Turbulent flows are characterized by fluctuations of the flow variables both in time and space; in view of the above-mentioned differences in time and space averaging, two analysis frameworks are discussed by means of practical examples. Concluding remarks are given at the end of the present contribution.

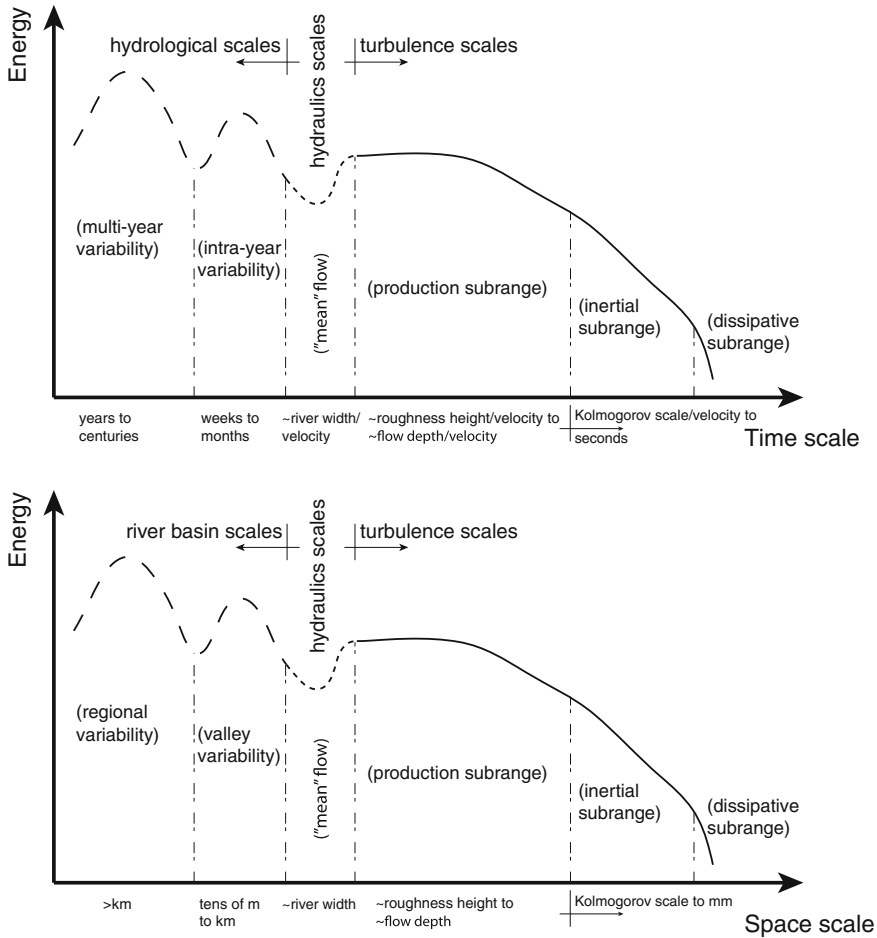
## 2.2 Background

### 2.2.1 *Typical Scales of River Flows*

River dynamics is characterized by a wide range of scales, which may vary from seconds to years (or even centuries), in terms of time, and from mm to km in terms of spatial dimensions. An engineer typically looks at scales from cm to km when addressing river flows, which may be related, for instance, to infrastructures constructed in the fluvial space or to the analyses of floods, whereas a biologist may be concerned with phenomena occurring at lengths of mm or less. In view of our analysis, in which temporal and spatial analyses complement each other, we inspect typical scales from both perspectives pertaining to the analysis of environmental physical processes in rivers. Figure 2.2, which is adapted from Nikora (2007), represents an interpretation of how the flow energy is distributed through temporal and spatial scales present in fluvial systems.

Tiny time scales, which are related to viscous diffusion and dissipation processes, are orders of magnitude smaller than seconds and are ubiquitous within the flow body. Floods with return periods of years, or even centuries, although less frequent, are responsible for important geomorphic processes, and drastically change the shape of the rivers. Regular floods in rivers (with small return periods, i.e., one to tens of years) destroy riverbed and bank armoring, feed the sediment continuum along the rivers and promote the regular biota renovation of the riverbed and floodplains. Although being less energetic events, secondary currents and flow cells typically generated locally, continuously shape the river morphology by their persistence in time. Turbulent coherent structures have time scales of seconds but they may transmit sufficient momentum to promote sediment entrainment and suspension.

The river basin scale (on the order of km) regulates the amount of water, sediments and organic matter arriving at a given river section. Furthermore, rivers can be regarded as open-channel flows with highly heterogeneous beds and irregular boundaries. These different geometric scales influence the flow structure and are thus related to the scales of the turbulence. Spatial scales simultaneously present in the fluvial milieu are due to grain roughness (grain-scale), bedforms (river width-scale), protuberant elements (grain scale to river width scale; this is especially important in low relative submergence flows), and channel configuration (valley scale, e.g., bends and braiding). Natural obstacles (e.g., riffles, pools, tree trunks, and root wads) and man-made structures (e.g., bridge foundations, groynes, and stream restoration structures) introduce further complexity by adding locally induced scales to the flow. Scales of orders of magnitude smaller than mm exist and are linked to micro-organisms and to molecular and viscous processes such as diffusion and energy dissipation. Mean advection is found at the larger scales of the rivers and is thus conditioned by the channel configuration. Turbulence generation is typically influenced by flow boundaries where the grain roughness, bedforms and



**Fig. 2.2** Distribution of the flow energy through time and spatial scales in river systems (adapted from Nikora 2007)

channel protuberances may play a role. Turbulent structures are, in turn, responsible for the continuous shaping of rivers and modify the roughness and geometry of the river boundaries.

The development of a theoretical model capable of solving such a variety of scales is nearly impossible. Restrictions, whereby a selective scaling that is limited to the necessary and sufficient detail of the problem under study, must be imposed. Examples of practical applications of the “restricted scale treatment” include Large Eddy Simulation (LES) models, where turbulence at large scales is resolved, whereas the smaller scales are modeled, and the Double-Averaging Methodology (DAM), which is a conceptual framework whereby up-scaling is performed in the

spatial and temporal sense, the flow is described at the largest spatial scales and the effects of spatial variability are included in the basic equations by means of dispersive-type terms (see Sect. 2.4.1).

### 2.2.2 Basic Equations

Conservation equations of flow properties, in addition to mass and momentum, include those corresponding to environmental variables such as diluted and undiluted substances, sediments and temperature. Within these equations, physical processes related to the turbulent nature of the flow affecting the conserved quantities are expressed mathematically: total convection, which is typically represented by a local (time partial) derivative and an advection term related to the mean advective field; diffusion, which corresponds to processes occurring at molecular and viscous scales and is represented by spatial gradients at very small scales; and turbulent transport (also called turbulent diffusion) corresponding to advection resulting from turbulent fluctuations. The co-existence of various scales of flow and flow pathlines in turbulent flows induces dispersion, which can be analyzed either in a spatial or temporal framework. Molecular diffusion, turbulent transport and dispersion contribute to the mixing efficiency of turbulence. Further processes such as sink and source terms, e.g., the gravity force which is typically the driving force of open-channel flows, or the interaction terms between different phases present in the fluid are also described by the basic equations.

In reference to turbulent flows, “ergodicity” refers to the realization invariance of the statistical properties of the flow, which is an essential condition to the application of the Reynolds decomposition. “Stationarity” refers to the temporal invariance of the statistical properties of a time-varying property, whereas “homogeneity” refers to the spatial invariance of the statistical properties of a spatially varying property. In other words, these three terms correspond to flow fields whereby the statistical properties of the turbulence are independent of the realization, time and position, respectively. Isotropic turbulence exists when the statistical properties are invariant, regardless of the direction considered for their analysis. Homogeneity and isotropy of turbulence are rarely found in nature but can be considered under given hypotheses and limited to several scales, hence enabling important simplifications in the analysis of the flows (Chassaing 2000; Pope 2000).

The Navier-Stokes equations, which correspond to the momentum equations of the flow, are likely adequate to describe the entire turbulence within a flow (Frisch 1995). Averaging the Navier-Stokes equations over the realization/time/space after application of Reynolds decomposition ( $\theta_j = \bar{\theta}_j + \theta'_j$ , where  $\theta$  is any generic flow variable susceptible to turbulence, the overbar stands for the turbulence-averaged component, and ' stands for the fluctuating component) produces the so-called Reynolds-Averaged Navier-Stokes equations (RANS), which, together with the mass conservation equation, for an incompressible although not necessarily homogeneous fluid, become

$$\begin{aligned}
& \underbrace{\frac{\partial(\bar{\rho}\bar{u}_j)}{\partial t}}_I + \underbrace{\frac{\partial(\overline{\rho'u'_j})}{\partial t}}_II + \underbrace{\frac{\partial(\bar{u}_k\bar{\rho}\bar{u}_j)}{\partial x_k}}_III + \underbrace{\frac{\partial(\bar{u}_k\overline{\rho'u'_j})}{\partial x_k}}_IV + \underbrace{\frac{\partial(\bar{u}_j\overline{\rho'u'_k})}{\partial x_k}}_V \\
& = - \underbrace{\frac{\partial\bar{p}}{\partial x_j}}_VI + \underbrace{\mu \frac{\partial^2\bar{u}_j}{\partial x_k\partial x_k}}_VII - \underbrace{\frac{\partial\bar{\rho}u'_k u'_j}{\partial x_k}}_VIII - \underbrace{\frac{\partial\overline{\rho'u'_k u'_j}}{\partial x_k}}_IX + \underbrace{\bar{\rho}g_j}_X
\end{aligned} \tag{2.1}$$

$$\frac{\partial\bar{u}_k}{\partial x_k} = 0 \tag{2.2}$$

where  $k$  and  $j$  are indices running from 1 to 3 and represent the three Cartesian spatial directions (1  $\equiv$  streamwise; 2  $\equiv$  spanwise; 3  $\equiv$  vertical);  $u$  stands for the velocity;  $x$  stands for the spatial coordinate;  $\rho$  stands for the density (time and space variant);  $t$  stands for time;  $p$  stands for pressure;  $\mu$  stands for dynamic fluid viscosity; and  $g$  stands for the gravitational acceleration. After the application of the Reynolds decomposition followed by realization/time/space averaging, and after comparing Eq. (2.1) to the Navier-Stokes equations written for instantaneous quantities, new terms appear in the momentum equation and introduce non-linearity; these new terms are related to the fluctuations in velocity and density. The terms in the momentum Eq. (2.1), which is also known as Reynolds equation, denoted using roman capital letters are as follows: *I*, local derivative of averaged momentum; *II*, local derivative of momentum fluctuation; *III*, mean advection of averaged momentum; *IV*, mean advection of momentum fluctuation; *V*, mean advection in the  $j$  direction of the momentum fluctuation; *VI*, averaged pressure term; *VII*, viscous diffusion term; *VIII*, Reynolds stress term; *IX*, turbulent transport of momentum fluctuation (turbulent diffusion); and *X*, average of the body forces term or, in this case, simply the gravity term.

The quantity  $\overline{u'_k u'_j}$  in RANS equations is the so-called Reynolds stress tensor, which is normalized by the density, and represents an additional stress in the flow compared to the Navier-Stokes equations. This non-linear term results from the Reynolds-averaging procedure of the Navier-Stokes equations. The elements in this tensor represent second-order moments of the fluctuating velocity field, normal moments or variances for the main diagonal elements and cross moments or covariances for the non-diagonal elements. The main diagonal elements ( $\overline{u'_k u'_j}$ , where  $k = j$ ) represent normal stresses along the three Cartesian directions ( $k = 1, 2, 3$ ) and shear elements from this tensor ( $\overline{u'_k u'_j}$ , where  $k \neq j$ ) correspond to shear stresses. The half trace of the Reynolds stress tensor per unit mass corresponds to the average turbulent kinetic energy (*tke*) of the flow (also per unit mass):  $\frac{1}{2}\overline{u'_k u'_k}$ . The anisotropy tensor is defined using the Reynolds stress tensor and is the basis for the analysis of turbulence states based on the Lumley triangle (Lumley and Newman 1977; Chassaing 2000; Jovanovic 2004; Dey 2014) (see Sect. 2.2.4).

The *tke* is extracted from the flow total energy ( $E$ ), which, for an incompressible flow, is essentially represented by the kinetic energy  $E \approx K = \frac{1}{2} u_k u_k$ . The mean kinetic energy (*mke*) of the flow is composed of a contribution corresponding to the kinetic energy of the mean flow  $\frac{1}{2} \overline{u_k} \overline{u_k}$  added to the *tke* ( $\frac{1}{2} \overline{u'_k u'_k}$ ). The Reynolds stress tensor and mean and turbulent kinetic energy (*mke* and *tke*) have well-defined transport equations, as detailed in Chassaing (2000), Pope (2000), among others. A dissipation term is present in all these equations and is dependent on the (mean or fluctuating) strain rate.

Empirical expressions describing the distributions of the Reynolds stress tensor components and of the *tke* are abundant in the literature and provide good results for uniform flows in hydraulically smooth beds (Cardoso et al. 1989; Nezu and Nakagawa 1993; Kironoto and Graf 1994). However, when the flow bed is hydraulically rough, which is typical of fluvial flows, the vertical distribution on the turbulence quantities is locally dependent of the bed forms below the height where the influence of the bed is felt (see Nikora and Smart 1997; Smart 1999; Nicholas 2001; Franca 2005b; Franca and Lemmin 2006b, among others). This inner region of the flow corresponds to the “roughness layer” (Nikora and Smart 1997).

Turbulent transport (or diffusion), as shown in term  $IX$  of Eq. (2.1), is difficult to determine experimentally; usually, it is either taken as negligible or modeled by a Fickian-type law. For homogeneous fluids, such as clear-water flow, the density is constant, and thus,  $\rho = \bar{\rho}$  and  $\rho' = 0$ , which considerably simplifies the transport equations. For a non-homogeneous system where the density differences in the fluid continuum are relatively small, the Boussinesq approximation may be used to simplify the hydrodynamic equations: the relative density variation is taken to be negligible when multiplying inertial terms and non-negligible when multiplying gravitational terms (Tennekes and Lumley 1972). Typically, the latter represents the buoyancy, which is the driving force of fluid movement due to density differences such as density currents (Simpson 1997).

For environmental applications, and in addition to momentum and mass conservation equations, other transport laws may be introduced for other environmental variables, such as the mass of substances, diluted or undiluted in the water and temperature (all these variables are turbulent variables and are susceptible to the Reynolds decomposition). Furthermore, the Reynolds-averaged transport equations for passive diluted and undiluted scalars (e.g., salt and sediments, respectively) may be written in terms of the volume fraction of the diluted species ( $\Phi_s$ ) or particle concentration of the undiluted species, respectively ( $c$ ).

When the species is diluted ( $\Phi_s$ ), the transport velocity corresponds to the fluid velocity. When the species is undiluted ( $c$ ), however, particles transported by the flow have their own momentum; thus, the advection field is given by the particle velocity ( $u^p$ ) rather than by the fluid velocity, which is generally different. The behavior of species particles can be assessed as a function of the Stokes number, which is defined as the ratio between the time scale of particles to react and the Kolmogorov time scale (e.g., Soldati and Marchioli 2009). “Massive particles”, or “inertial particles”, are characterized by a Stokes number of order one or larger,



whereas “water particles” are characterized by a vanishingly small Stokes number. The Stokes number significantly influences particle suspension and deposition; for example, large Stokes number particles that possess sufficient momentum may either coast through some accretion region and deposit by impacting directly at the flow bed or go into suspension. Otherwise, those particles that have not received sufficient momentum are forced to deposit on the bed due to gravity and inertia.

Species diffusion is typically modeled as a Fickian process, in practice by  $\Gamma_m \frac{\partial^2}{\partial x^2}$ , where  $\Gamma_m$  is a molecular diffusivity. Turbulent transport or diffusion may also be modeled as a Fickian process with  $\Gamma_t$ , which is the so-called turbulent diffusivity. In Fickian processes the property flux travels from regions of higher concentration to regions of lower concentration, proportionally to the concentration gradient. For a non-conservative substance (for instance suspended sediments that can deposit or erode from the riverbed), sink or source terms must be added when the integration of the species conservation equations is made.

Further transport equations pertinent to the analysis of environmental turbulent flows, such as those in terms of pressure, fluctuating momentum, Reynolds stress tensor, kinetic energy (mean, turbulent and instantaneous), species fluctuation and variance, and energy dissipation (cf. Monin and Yaglom 1971; Frisch 1995; Chassaing 2000; Pope 2000, among others), although not shown here, may be derived from these basic equations.

### 2.2.3 Energy-Based Description of Processes

As stated in Sect. 2.1, the energy transfer for which turbulence is responsible generally evolves through a so-called cascading process from the larger scales, which extract energy from the total flow, thus generating turbulent kinetic energy, to very small scales, where viscous dissipation takes place; the kinetic energy of the flow is eventually lost as heat. At the intermediate scales, an eddy fragmentation process occurs with minimal energy transfer; only break-up of the large scales into smaller ones is observed without substantially influencing the energy content of the flow. The scales of the intermediate eddies corresponding to this transfer belong to the so-called inertial sub-range (in contrast to the productive and dissipative sub-ranges for large and small eddies, respectively). The first description of the energy cascade process of turbulent flows was provided by Richardson in the 1920s and was later formalized and included in the papers on the theory of turbulence published by Kolmogorov in 1941 (Frisch 1995). Since then, much work has also been performed on nonlocal turbulence processes, in which eddies of sizes within the available ranges significantly interact (e.g. Nazarenko and Laval 2000). However, the authors are not aware of specific studies dedicated to riverine flows.

At the flow solid frontiers of open-channels, due to the non-slip boundary condition at the river bed and banks which are fixed (non-moving), the velocities are zero. Due to viscous effects, a thin layer exists near these frontiers where

velocities are very small. Here, the flow is consequently laminar and this region of the flow is called the laminar or viscous layer (cf. Chassaing 2000). Important velocity gradients exist and generate unstable flow conditions, where the generation of turbulent eddies is promoted. Therefore, bounded flows have one main source of turbulence generation in their solid boundaries. Furthermore, the presence of solid boundaries protruding in the flow typically induces flow separation and, consequently, recirculation cells in shaded areas. Again, strong gradients between the lee of these protuberances and the external flow produce new turbulent structures that may be advected downstream, thus conditioning the flow structure. The size of the eddies generated by individual boundary protuberances scales, typically, with the dimension of the protruding elements. Examples of boundary singularities and irregularities in rivers causing eddy generation include sediment clusters or boulders protruding from the riverbed [submerged (Buffin-Bélanger and Roy 1998; Buffin-Bélanger et al. 2000; Franca 2005b) or emerging (Tritico and Hotchkiss 2005)], high relative roughness riverbeds (Baiaomonte et al. 1995; Katul et al. 2008), vegetation patches (Tanino and Nepf 2008; Siniscalchi et al. 2012; Sukhodolova and Sukhodolov 2012; Ricardo 2014), wood debris or remains of organic elements (Shields et al. 2004; Blanckaert et al. 2014).

The wide range of the above-mentioned turbulent eddies are macroscale coherent structures belonging to the productive range. Coherent structures are formed, typically, in shear zones such as the bottom boundary layer and depth transition layers, and their generation is due to the interaction between regions with different momenta. They have a recognized role in the mechanisms of sediment entrainment in river flows (Séchet and Le Guennec 1999; Cellino and Lemmin 2004). At rough boundaries, which in rivers are typically rough sand or gravel beds and banks, dune and ripple fields, subaquatic plant canopies, or vegetation patches, a continuous shear zone is imposed on the flow, and periodical generation of coherent structures is observed, corresponding to a bursting process.

Chassaing (2000) introduced the concept of physical coherence together with statistical coherence. The physical coherence concept is related to the fact that turbulent structure properties (for example, geometric, kinematics and dynamics) change relatively slowly with respect to their representative scale domain; turbulent structure properties are, consequently, conveyed by the flow field. Coherent structures have a short life cycle; they cannot be identified with a time-averaged analysis and require an investigation based on time and space correlations or on flow visualization techniques. In laminar and transitional flows, coherent structures occur periodically, whereas in turbulent flows they occur chaotically in space and time. Hence, in addition to flow visualization techniques, conditional sampling and statistical techniques have to be used in the detection and characterization of coherent structures. Large- and small-scale coherent motion within the coherent structure range can still be distinguished. Large-scale motion typically scales with the boundary layer thickness (eddies scale with the flow depth in open-channel flows or with the depth-transition region in compound channels), whereas small-scale motion scales with parcels of the boundary layer (eddies scale with dimensions comparable to the roughness sizes). Each of these groups acts differently but possibly jointly in the flow energy balance.

Large-scale flow structures and secondary currents are characterized by their quasi-steadiness in time; however, they introduce 3D complexity to river flows. They are associated with mesoscale features (river depth and width) in the river geometry such as bends, meanders, floodplains and section changes. Large-scale flow structures can also be defined in the time frame and include waves and pulses traveling in the flow and interacting with the turbulent field.

### ***2.2.4 Frameworks for Turbulence Analysis***

The Eulerian and Lagrangian descriptions of the flows are equally valid for the derivation of the hydrodynamic equations and are easily related through the velocity field. However, the Eulerian approach is the most common in practice (Currie 1993). Both descriptions are associated with different techniques of flow observation, the Eulerian corresponding to an approach wherein the fluid variables (velocity, pressure, etc.) are measured continuously in time at one or more fixed points and the Lagrangian corresponding to an approach where the fluid variables are followed along the flow trajectories (i.e., the positions of particles or of diluted substances as a function of time). While Lagrangian techniques naturally provide a measure of dispersion and diffusion in turbulent flows, from a practical point of view the Eulerian techniques are simpler and widely used (cf. Romano et al. 2007; Ghilardi et al. 2014 for reviews of measurement techniques of turbulent flows and rivers).

Time and space scales of turbulence are often linked through the Taylor frozen-turbulence hypothesis by means of an advection velocity. This permits the linking, for instance, of the partition of the turbulent energy through scales pertaining to both dimensions. However, to apply Taylor's hypothesis and thus translate Eulerian observations in time into the space domain, the turbulent eddies are taken to be undeformable by the transport imposed by the mean and turbulent velocity fields (Chassaing 2000). Thus, generally, two analytical frameworks for the study of turbulent river flows pertaining to the two dimensions (time and space) are needed. Because time and space averaging do not commute for nonlinear processes, both are generally used independently as functions of the process of interest.

The technique based on anisotropy invariants proposed by Lumley and Newman (1977) (see, also, Chassaing 2000; Jovanovic 2004; Dey 2014 for further details) provides a methodology of classifying the anisotropy degree and nature of turbulence, which is irrespective of the reference system. The so-called Lumley triangle technique (or Lumley plots), which contains any turbulence state within its limits, enables the identification of the state of turbulence being studied in the flow regions in an analysis: 3D isotropic turbulence, 2D isotropic turbulence, and 1D turbulence. In the transition between these limiting states of turbulence, two characteristic types of turbulent structures are found: pancake-shaped turbulence, which corresponds to a situation where two of the fluctuation components are equal and considerably larger than the third, and cigar-shaped structures where two of the turbulence

components, and hence the normal stresses values, are equal and the third has a substantially higher value. The analysis based on the Lumley plots, combined with other observational techniques, such as the quadrant analysis (Lu and Willmarth 1973; Nakagawa and Nezu 1977), permits the establishment of a proper framework for the analysis of turbulence and forms the basis of useful simplifications for the analysis of complex 3D flows (Mera et al. 2014). The knowledge of the nature of turbulent eddies in river flows allows the establishment of restricted spatial frameworks for the analysis of these flows, as will be seen in Sect. 2.4.

## 2.3 Time-Frame Analysis

### 2.3.1 Steady Flows

As mentioned in Sect. 2.2.2, stationarity or steadiness implies temporal invariance of the flow statistical properties; however, this enables spatial variations. In formal terms, and considering the property “mean momentum” as an example, this means that the local derivative in Eq. (2.1) (term  $I$ ) does not exist, thus considerably simplifying the practical resolution of hydrodynamic equations.

The concept of stationarity is related to the time scale of the problem and to the statistical property under analysis. Considering a given time-scale and a statistical property, we consider the flow stationary when the value of this statistical property is stable, i.e., it does not change with the increase of the time-scale. This stationarity is conditioned to the statistical property under analysis; in the study of first-order moments (means), the time scale where these are stable is smaller than when considering higher order moments. Common statistical properties used in the analysis of turbulent river flows are the Reynolds stress tensor and the *tkc* (based on second-order statistical moments). For these properties, the statistical invariance (defining an appropriate stationarity) is less demanding in terms of the time scale compared to, for instance, analyzing turbulent transport (third-order moment), as in the term  $IX$  of Eq. (2.1).

In the practical analysis of natural floods in a limited river reach, engineers commonly consider a constant peak flood discharge for flow modeling, which may last for several hours. This assumption of constant discharge is valid if the time response of the reach under analysis to flow changes is less than the duration of the peak discharge, i.e., if the travel time of the flood wave along the reach is smaller than the latter and if there is no considerable spatial variation in the discharge. However, in the analysis of an entire river basin, the time response of the reach, which may be comparable to the concentration time of the catchment, is not compatible with a typical flood duration; thus, stationarity can no longer be used.

The theoretical advancements in the understanding of complex 3D flows by physical modeling are typically made considering steady-state conditions. The stationary approach is valid and, sometimes, is the best method to study fundamental aspects of turbulent flows in an isolated fashion. The spatial characterization of

turbulent quantities obtained under steady-state flow conditions allows identifying, quantifying and understanding the mechanisms of the transport of undiluted or diluted species (including dispersive and diffusive processes) and of the processes of erosion and deposition which are related to the shaping of the river morphology.

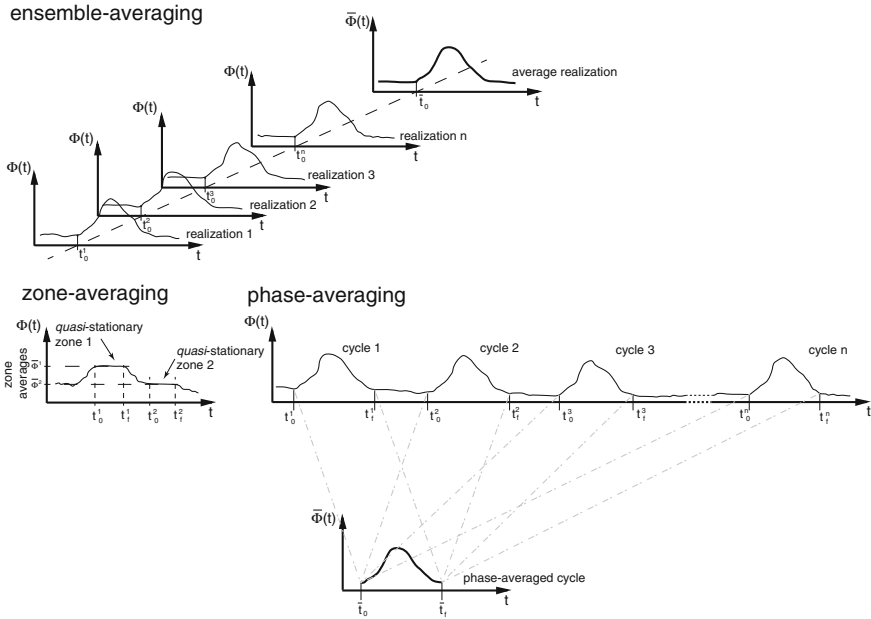
Under steady-state conditions, the determination of turbulence quantities, such as the Reynolds stresses [term *VIII* of Eq. (2.1)], is straightforward (Chassaing 2000). The turbulent structure of the flow in complex geometries and the interpretation of the related fluvial processes are determined through the analysis of the spatial distribution of the mean velocity and vorticity fields and of the second- and third-order moments present in the conservation equations, provided that the data is sufficient to obtain stable statistical moments. Leite Ribeiro et al. (2002), Proust et al. (2013), Ricardo et al. (2014) are examples of recent theoretical analyses of fluvial turbulent processes with highly complex boundary geometries performed by assuming steady-state flow conditions. When performing field studies, which are typically based on measurements of instantaneous velocities, researchers pay special attention to the steadiness of the flow to assure that measurements made in multiple spatial positions are representative and can be combined in a time-averaged description of the flow (Franca 2005a; Saggiori et al. 2012).

### 2.3.2 Unsteady Flows

When unsteadiness can no longer be ignored, i.e., when the time scale under analysis is larger than that of the mean flow variations, full Reynolds equations have to be considered. In unsteady flows, the inference of statistical moments may be made by means of different types of average operators (for their definition, cf., for instance, Chassaing 2000).

If in a turbulent flow the processes are ergodic, i.e., the flow statistical properties are invariant with their realization, the ensemble average allows proper estimates of turbulent properties of the flows. This consists of taking the average of all the possible realizations of one turbulent flow (Pokrajac and Kikkert 2011) and it is applied to transient processes in river flows such as wave passage, bore passage, dam-break flows, breaking waves, and ship waves (cf., Fig. 2.3).

In transient phenomena, a time interval may exist wherein the flow properties remain stationary, i.e., where statistical properties are invariant for some duration. The level of invariance of the statistical properties depends on the flow turbulence level and on the time interval: for instance, we may be able to obtain stable time-averaged quantities and stable quantities derived from second-order moments (Reynolds stresses, *tk<sub>e</sub>*, etc.) but not stable higher statistical moments, hence the designation of *quasi-stationary* flows. For these cases, we may apply a zone or conditioned average along the *quasi-stationary* period to infer turbulent properties: recent examples of the application of zone averages to riverine flows include (Aleixo 2013), who used this approach for the statistical analysis of dam-break



**Fig. 2.3** Illustration of the type of averages that can be applied to turbulent signals: ensemble average, zone average and phase average

flows based on PIV high-frequency measurements, and (Nogueira 2014), who applied zone averages to obtain mean velocity profiles and Reynolds stresses in a lock-exchange density flow based on PIV measurements.

In the case of a periodic phenomenon, an average may be obtained over different cycles, with each being comparable to a realization of the phenomenon, to estimate turbulent properties; this is called phase (or also composite) averaging. Such an average is adequate for the analysis of time/space flow features which have a limited life cycle, and pseudo-periodicity is verified (pseudo-periodic cycles do not necessarily have a constant period and are not necessarily consecutive). Conditional sampling and statistical techniques have to be used in the detection and characterization of the cyclic processes. Techniques used for the detection of periodic events in open-channel flows include quadrant threshold analysis (Willmarth and Lu 1972; Nakagawa and Nezu 1977, among others), wavelet decomposition (Foufoula-Georgiou and Kumar 1994; Yoshida and Nezu 2004; Franca and Lemmin 2006a, among others), proper orthogonal decomposition (Berkoos et al. 1993; Holmes et al. 1998) and empirical mode decomposition (Huang et al. 1998; Franca and Lemmin 2014). Examples of the application of phase averages for obtaining statistical properties of turbulent flows include the following: Franca and Lemmin (2006a), where the identification and reconstruction of coherent structures was performed based on wavelet multi-resolution analysis for field measurements; Nogueira et al. (2014), where a cyclic pattern of growth and a decrease in the head

of density currents was described and quantified in terms of kinematics and dynamics; and Franca and Lemmin (2014), where large-scale coherent structures were detected using Huang's empirical mode decomposition (Huang et al. 1998) and reconstructed with phase-averaging techniques based on a Hilbert transform of the velocity signal (Huang et al. 1999).

Recently, several authors have developed methods to address upscaling techniques for flows with time-varying boundaries: Pokrajac and Kikkert (2011) presented the Reynolds-Averaged, Depth-Integrated Navier-Stokes equations applied to an upsloping transient bore where the water surface was not constant and contained air bubbles, and Nikora et al. (2013) presented double-averaged conservation equations for mobile-boundary conditions, namely, mobile rough beds. The range of application of these techniques is wide in the field of fluvial flows with time intermittent boundaries such as the mixing layer of density currents reproduced in laboratory (Lopes et al. 2013).

## 2.4 Space-Frame Analysis

### 2.4.1 Homogeneous and Heterogeneous Flows

Further simplifications of the hydrodynamic equations can be made by assuming the uniformity of the flow, implying that a spatial variation of flow velocities does not exist,  $\frac{\partial}{\partial x_k} = 0$ . A practical result of this simplification that is most frequently used by engineers is the relation between the bed shear stress and the slope of an open-channel, which, for a one-dimensional flow in a wide channel with rectangular cross-section, is  $\tau_b = \rho ghS$  ( $h$  is the water depth, and  $S$  is the longitudinal bed slope of the channel). This relation is easily demonstrated when considering the one-dimensional integral form of the momentum conservation equation (Currie 1993) over a control volume ( $V_c$ ) with surface  $S_c$ . For uniform flows, all terms become zero except the streamwise component of the gravitational force ( $\int_{V_c} \rho g_1 dV$ ) and the bed shear force term ( $\int_{S_c} \tau_b dS$ ), which must equilibrate, thereby resulting in the above expression for the determination of  $\tau_b$ . In other words, the force corresponding to the streamwise component of the weight of the fluid inside the control volume (gravity-driven force) is balanced by the bed shear stress resistance force. This assumption thus requires the use of strong arguments, and it is commonly misused in highly varied flows such as mountain rivers or watercourses with complex geometries.

Although hard to find in natural rivers, uniform flows are commonly used to experimentally or numerically study turbulent flows. The empirical closures for the vertical distribution of the Reynolds stress tensor and *tke* in open-channel flows mentioned in Sect. 2.2.2 and that have been established and accepted for some decades are an example of this assumption.

Typically, natural rivers have boundaries which are very spatially varied and heterogeneous. For instance, for low relative submergence flows, i.e., for small values of  $h/D$ , where  $D$  is a geometric parameter that is representative of the bed roughness, there is no self similarity within the lower regions of the flow, especially within the troughs and crests of the riverbed. Here, the flow is highly 3D and heterogeneous, and accounting for spatial variability effects with the Reynolds equation becomes complex. To smoothen flow irregularities, a spatial averaging operation may be applied to the RANS equations, thus resulting in the so-called Double-Averaged (in both time and space) Navier-Stokes (DANS) equations (Nikora et al. 2007a). If averaging is performed at sufficiently large scales to smoothen the flow spatial heterogeneity, then uniformity may be assumed simplifying the hydrodynamic equations. The scale over which upscaling is performed should be sufficiently large to ensure that the statistical properties do not depend on the averaging volume anymore; thus, the flow may be treated as homogeneous. The variability of the flow at scales that are smaller than the upscaling domain is considered by means of additional (dispersive or form-induced) components that are incorporated into the DANS equations when these are derived. These additional terms characterize momentum and scalar fluxes in the flow regions under the influence of the heterogeneous boundaries and where flow similarity is not possible. In river engineering, it is often desirable to estimate the vertical distribution of streamwise flow velocity and momentum balances (including the determination of stresses and drag forces) based on simple assumptions made on scales that are sufficiently large to incorporate the smaller scale phenomena.

Given the high spatial variability of the flow characteristics in gravel-bed rivers with low relative submergence, the application of the double-averaged methods (DAM) using a minimum scale of one wavelength of the bedforms (Raupach and Shaw 1982) seems appropriate. The analysis and modeling of heterogeneous and irregular-bounded open-channel flows by means of double-averaging (DA) methods is presented by several authors for different problems in fluvial hydraulics (Aberle and Koll 2004; Manes et al. 2007; Franca et al. 2008; Stoesser and Nikora 2008; Ferreira et al. 2010, among others). Franca et al. (2010) and Mignot et al. (2009) present results of the application of double-averaging methods to the streamwise velocity and Reynolds stresses of gravel-bed open-channel flows.

Recently, a journal's special issue on the application of the double-averaging approach to rough-bed flows was published (cf. Nikora and Rowinski 2008), and an overview of its application to geophysical, environmental and engineering physics was given. A comprehensive overview of the methodology and its applications to environmental hydraulics was also recently provided in Nikora et al. (2007a, b). Spatial averaging in the context of the DA technique is commonly performed along horizontal surfaces parallel to the riverbed, and the main formalisms described in the literature refer mainly to this type of application. Similar upscaling methods may be applied to any heterogeneous boundaries of the flow, such as river banks or vegetation canopy, the latter constituting an open boundary between different regions of the flow.



### 2.4.2 Turbulence Evolving on the Vertical Plane

In wide rivers, the boundary layers produced by the banks are usually neglected because they occupy a small region of the flow; thus, if sufficiently wide, open-channel flows and the physical interpretation of energetic turbulent processes are treated as 2D (in the vertical-longitudinal plane,  $x_1 - x_3$ ) along the water column. This applies to existing closures for the distribution of mean flow quantities and of turbulence-related quantities such as Reynolds stresses, *tke*, turbulent scales and related production, diffusion and dissipation terms (Nezu and Nakagawa 1993; Chassaing 2000; Pope 2000). Hence, these distributions depend only on the lower and upper boundaries of the flows.

In line with the consideration of a 2D plane is the analysis of energetic events that contribute to the production of Reynolds shear stress in the flow. The bursting phenomenon results from a class of coherent structures occurring mainly in the near-wall region. This results from a quasi-cyclic process that is composed of interactions in the four quadrants of a 2D Cartesian longitudinal framework, hereafter called shear events: outward interaction, ejection, inward interaction and sweep (Nezu and Nakagawa 1993). Shear events cannot be identified with time averaged analysis and, in addition to time and space correlation measurements and flow visualization techniques, require conditional sampling and statistic techniques for their detection and characterization. Important works on the analysis of turbulence events contributing to Reynolds stress production include Antonia and Atkinson (1973), Corino and Brodkey (1969), Grass (1971), Kim et al. (1971), Kline et al. (1967), Nakagawa and Nezu (1977) and, more recently, Hurther and Lemmin (2000) and Adrian et al. (2000). Nakagawa and Nezu (1977) made a prediction of the contribution of all types of shear events to the production of Reynolds stresses. To quantify their results, they used the quadrant threshold method developed by Corino and Brodkey (1969), Lu and Willmarth (1973), Willmarth and Lu (1972).

The role of the so-called vertical turbulence is the most important in terms of sediment mobilization and transport (Séchet and Le Guennec 1999; Cellino and Lemmin 2004). In particular, knowledge of the turbulent pressure fluctuations leading to drag and lift forces at the water-sediment interface is fundamental. Recent measurements performed with miniaturized piezoresistive pressure sensors, which are small and include high sensitivity and accuracy, have provided significant information on turbulent forces on single grains at river beds (Detert et al. 2010). One aspect that attracts considerable attention is the rate at which bed sediment is entrained by turbulent shear flow. The recent contribution of Zhong et al. (2011) attempted to model such an entrainment in terms of kinetic theory for multiphase flows. This has advantages over other theories in terms of (a) accounting for the influences exerted on the sediment by external forces and (b) providing a statistical description of the random motion of sediments due to turbulence. These advantages enable the kinetic theory to act as a bridge between microscopic and macroscopic scales of moving sediment particles.

Experimental studies were also performed to quantify the near-bed turbulence characteristics for fluvial flows under sediment entrainment limit conditions and in the presence of mobile beds. Gyr and Schmid (1997), using Laser Doppler Anemometry (LDA) measurements over a smooth sand bed, observed that the presence of intense intermittent sediment transport increases the extreme values of shear stress while the flow becomes more organized in the second and fourth quadrants, mainly increasing the importance of sweep events to turbulence production. The period between events in the second and fourth quadrants decreases considerably in the presence of sediment transport, thereby producing more frequent ejection and sweep events. More recently, Dey et al. (2011) found that the observed time-averaged streamwise velocity is further from the logarithmic for immobile beds than for entrainment-threshold beds. They also found, by means of a quadrant analysis, that in the near-bed flow zone, ejections and sweeps in immobile beds cancel each other, thereby giving rise to the outward interactions, whereas sweeps are the dominant mechanism causing sediment entrainment. Finally, the bursting duration for entrainment-threshold beds is smaller than that for immobile beds; in contrast, the bursting frequency for entrainment-threshold beds is larger than that for immobile beds. Santos et al. (2014) showed that, generally, the sediment transport of sand decreases the transported momentum and maximum shear stress values but increases their frequency of occurrence in time. The analysis of the probability distribution function of both ejections and sweeps shows an effect of sediment transport in terms of the reduction in the frequency of large events and in the increase of the frequency of small events. This may be due to the breaking of eddy coherence by sediment motion and is especially observed in the so-called pythmenic region (Ferreira et al. 2012).

Prandtl's first and second types of secondary flows which are perpendicular to the streamwise direction of the flow, have thus expression in vertical planes, transverse to the main flow direction (cf. Nezu and Nakagawa 1993; Nikora and Roy 2012). These secondary flows are divided into two classes: the first type of Prandtl's secondary flow is observed when the streamwise mean vorticity is enhanced by vortex stretching, occurring typically in river bends and meanders. The second type of secondary flows occurs due to turbulence heterogeneity, and no curvature of the principal flow direction is required to exist. Secondary current cells may have a signature in the river bed morphology and may be influenced in turn by the channel roughness (Tsujimoto 1989).

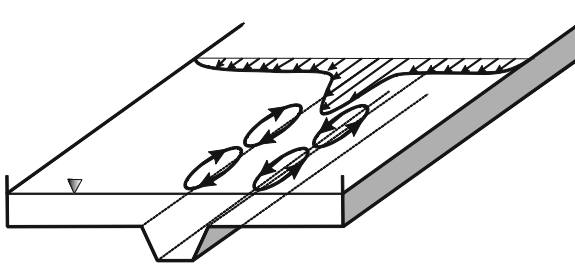
The vertical distribution of turbulence has also a significant interaction with the instream vegetation which in turn conditions the river morphology (e.g., Neary et al. 2012). Compared to unvegetated flows with the same discharge and slope, vegetation causes flow blockage and increases channel resistance. If the vegetation is submerged, the consideration of a 2D vertical-longitudinal plane on the analysis of the flow is still valid as a canopy-like flow occurs (Nepf 2012). In vegetated flow where the vegetation is emergent, three regions of the flow may generally be observed, whereby the flow is controlled by the riverbed and the vegetation (lower region), solely by the vegetation or by the vegetation and the free surface (upper region). The middle region is generally self-similar as concerns turbulent quantities.

### 2.4.3 Turbulence Evolving on the Horizontal Plane

Natural geomorphological features and man-made obstacles opposing the flow cause geometrically-induced gradients in river flows, thereby inducing coherent vortical structures with vertical axes. These structures may range from large-scale, slowly evolving eddies in regions of recirculation and flow stagnation to energetic shear layers, whirlpool-type vortices, and curvature-induced streamwise vortices. The ability to reproduce the influence of such vortices is fundamental for a number of theoretical and practical issues. From a theoretical point of view, the understanding of lateral transfer of momentum and scalar (for instance, in compound channel flows) are current topics of research. From a practical perspective, large-scale vortical structures with vertical axes may influence the design of bridge foundations resistant to scouring and the stabilization of stream banks.

Two main, complementary, approaches can be used to describe the generation of large-scale vortices in a river characterized by topographic bed changes. On the one hand, vortical structures can be regarded as the manifestation of the shear instability at the junction of two different streams (van Prooijen and Uijttewaal 2002; van Prooijen et al. 2005); on the other hand, they can be seen as the outcome of differential energy dissipation of shallow-water currents interacting with submerged obstacles (Brocchini et al. 2004; Soldini et al. 2004; Kennedy et al. 2006).

To discuss the role of large-scale eddies in association with topographic bed changes we refer to the simple case of the compound channel of Fig. 2.4, which shows macrovortices in the transition regions and a mean flow velocity distribution along the transversal direction (see Stocchino et al. 2011). Recent experimental investigations (Stocchino and Brocchini 2010), which were based on use of the PIV technique, revealed that the population and properties of macrovortices largely depend on the typical depth gradients, i.e., on the ratio  $r_h$  between the largest and smallest flow depths. Shallow flows ( $r_h > 3$ ) are dominated by strong shearing and large macrovortices populate the transition region between the main channel and the floodplains. The mean streamwise velocity induced by intermediate flows ( $2 < r_h < 3$ ) is characterized by a dip in the transition region, while it closely resembles that occurring in a rectangular channel in the case of deep flows ( $r_h < 2$ ).



**Fig. 2.4** Sketch of a typical compound-channel geometry with mean flow profile and large-scale vortices at depth transition

For both the latter cases, the shear in the transition region decreases, and the macrovortices are also generated in the wall boundary layer of the floodplains.

The visualization of macrovortices can be performed using typical indicators such as the Hua and Kline eigenvalue  $\lambda_+$  of the local acceleration tensor (Hua and Kline 1998). An illustration of such a population is reported in Fig. 2.5 for the cases of shallow (top) and deep (bottom) flows. Shallow flows are characterized by large quasi-2D vortical structures with vertical axes, which behave as organized domains with distinct dynamical roles and dominate turbulence production; these flows are generally resolved considering 2D depth-averaged models. Under deep flow conditions, fewer macrovortices can be recognized, horizontal turbulence production becomes less prevalent, and 2D streamwise-spanwise approaches are no longer valid.

The mixing induced by these dynamical domains can be described in terms of both absolute (single-particle) and relative (e.g., particle pairs) statistics (e.g., Provenzale 1999; LaCasce 2008; Stocchino et al. 2011). Typically, under shallow flow conditions, macrovortices strongly influence the growth in time of the total absolute dispersion after an initial ballistic regime after their formation, leading to a

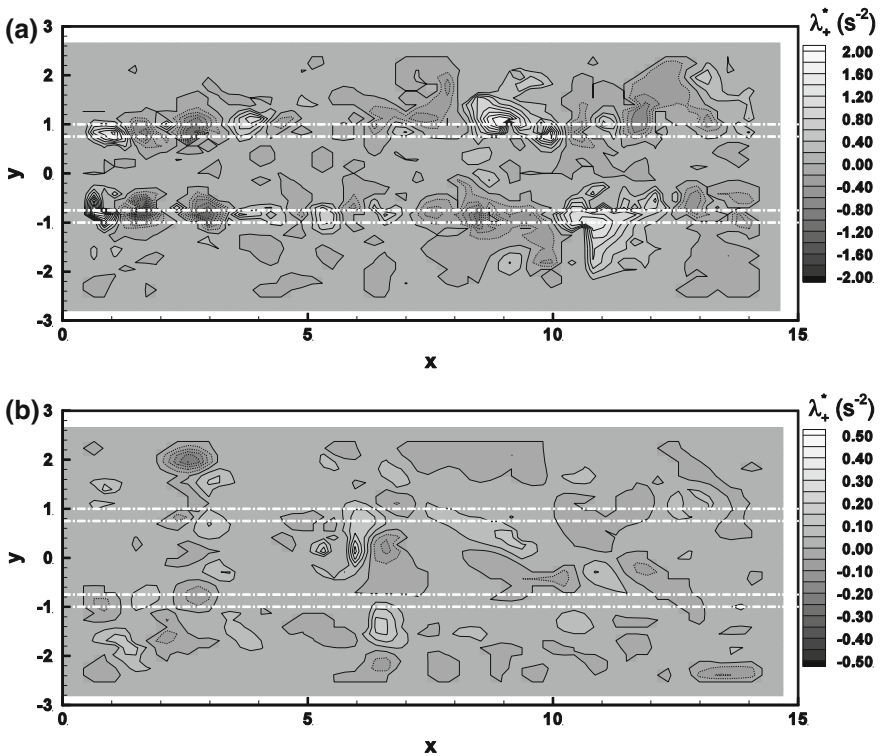


Fig. 2.5 Examples of 2D maps of the positive Hua-Kline eigenvalue. *Top* shallow flow conditions. *Bottom* deep flow conditions. The *dotted white lines* indicate the transition regions of the compound channel. Adapted from Stocchino et al. (2011)

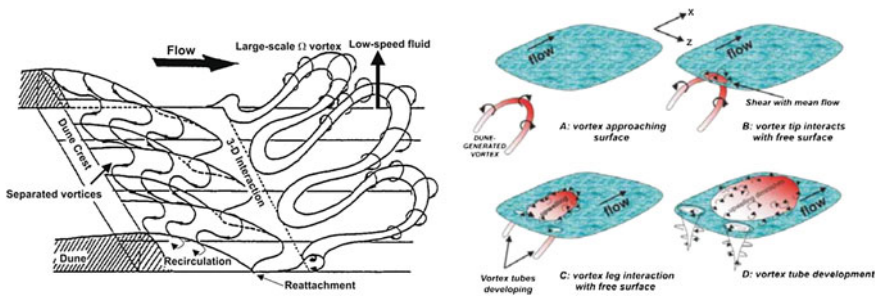
non-monotonic behavior. Under deep flow conditions, on the contrary, the absolute dispersion exhibits a monotonic growth because the generation of transitional macrovortices does not occur. In all cases, an asymptotic diffusive regime is obtained. Multiple-particle dynamics are controlled by the ratio between the largest and smallest flow depths,  $r_h$ , and by the Froude number. Different growth regimes of the relative diffusivity exist as a function of the flow conditions. This is associated with different energy transfer processes, which show a different asymptotical shape as a function of the separation scales and the Froude number. An equilibrium regime is observed by analyzing the decay of the finite-scale Lyapunov exponents with the particle separations.

Further investigation on compound channels, namely, on the distribution of turbulent quantities and on energetic processes related to 2D macrovortices present at the interface between the main-channel and floodplain flows, include Bousmar and Zech (1999), Kara et al. (2012), Knight and Shiono (1990), Proust et al. (2013), Tominaga and Nezu (1991), van Prooijen et al. (2005).

As introduced earlier in Sect. 2.4.2, when the vegetation is emergent, a middle region of the flow exists and is generally controlled only by the vegetation, eventually becoming self-similar in terms of turbulent quantities. Here, the vegetation conditions local velocities, turbulence intensities, turbulent Reynolds stresses and their vertical and horizontal distributions (e.g., Nepf 1999). Ricardo et al. (2014) estimated the terms in the *tke* transport equation in a flow with emergent arrays of cylinders by considering velocity measurements in horizontal planes. With specific reference to the role of vegetation in turbulent flows evolving in compound channels, Koziol (2013) found, on the basis of dedicated experimental investigations, that trees placed on the floodplains do not significantly change the values of the relative turbulence intensity in the entire compound channel, but they do change the vertical distributions of the relative turbulence intensities in the three components in the floodplains and over the bottom of the main channel.

#### ***2.4.4 Turbulence and Vorticity Evolving from Vertical to Horizontal***

A substantial amount of literature is devoted to the generation of horseshoe or hairpin types of vortices, in turbulent flows over plane boundaries (Chassaing 2000). The phenomenology associated with these vortices is somehow related to the experimental and field investigations of sand waves, which have documented turbulent events called “kolks” and “boils” (Matthes 1947; Coleman 1969); these events have a 3D signature in the flow, evolving from a 2D vertical-horizontal structure in a first stage. The so-called “kolk-boil” mechanism is one of predominant turbulent events occurring over sand waves in fluvial, estuarine, and marine coastal environments (Ha and Chough 2003). A kolk is a slowly rotating, upward-tilting vortex on the stoss face of a sub-aqueous bedform. A strong kolk may reach the water surface, create a cloudy columnar sediment-fluid mixture, and form a



**Fig. 2.6** *Left* model of vortex topology associated with dunes proposed by Nezu and Nakagawa (1993). *Right* model of flow illustrating the interaction of dune-related macro-turbulence with the water surface proposed by Best (2005). Adapted from Best (2005)

raised circular or oval patch at the air-water interface, which is referred to as a boil (Matthes 1947; Kostaschuk and Church 1993). These boils are created on a scale that is comparable to the flow depth as a first approximation.

These vortical structures are generated by a complex mechanism that is initiated at the reattachment point at the lee side of an obstacle, e.g., sand ripples and sand dunes (Nezu and Nakagawa 1993). Best (2005) proposed a model for the stages of interaction of a vortex loop with the surface and for the manner by which the phenomenon is manifested as different upwelling motions as the boil evolves and erupts on the surface (see Fig. 2.6). This model shows how the initial transverse vorticity is accompanied by vertical vorticity as the boil evolves and as the vortex legs of the vortex loop attach to the surface; this pattern is common in many natural rivers. It is important to note that Best (2005) also argues that this upwelling and flow surface interaction must induce subsequent downwelling toward the bed to satisfy flow continuity.

Flow in compound meandering channels is an example of the nature of tridimensional flows, combining both complex flows in a compound channel (with 2D vortical structures with a vertical axis) and in a meandered channel (with secondary flow cells with a streamwise axis, developing in a plane transverse to the flow). Experimental studies that focus on compound meandering morphologies are scarce (Shiono and Muto 1998; Shiono et al. 2008). Mera et al. (2014) characterized the hydrodynamics and turbulence patterns in a real compound meandering channel using the anisotropy invariants combined with quadrant analysis techniques.

## 2.5 Conclusions

Many textbooks are available on the basic theoretical aspects of turbulent flows and on their inclusion in fluid mechanics studies. Some examples of useful books are referenced in the present contribution, which focuses on specific aspects related to

turbulent riverine flows. Although not exhaustive, the theories discussed herein were chosen based on their relevance to the study of fluvial processes, which means that the present paper is organized in terms of methodological approaches used to address turbulent fluvial flows, rather than in terms of phenomenological aspects. Furthermore, themes that are usually subject to misconceptions and erroneous interpretation by engineers and researchers studying river flows are discussed.

One of the main issues explored here is the notion of scales and their selective use. River flows are characterized by a large variety of scales (time or space scales). Using both temporal and spatial frameworks, we show that the concepts of ergodicity, stationarity and homogeneity depend on the relation between the typical time or space ranges of the phenomenon being studied and on the scale of turbulence evolving within this.

In addition, the focus of the analysis strictly depends on the scales of the phenomenon, which is illustrated in terms of the demands of the higher order statistical moments being studied. For example, the analysis of the mean flow requires a time range that is smaller than that needed by second-order analyses, where Reynolds stresses and *ike* may be included.

Only a partial overview of the basic equations for the study of turbulent flows is given. However, references of exhaustive formal analyses, as well as a discussion of sources of information on other transport equations of variables important for river flows, are provided. Topics related to mass, momentum and species conservation are also proposed. The latter topics are particularly important in terms of the environmental analysis of rivers, where the transport of species, diluted or not, is important. Problems of pollutants or salt mixing, as well as sediment transport, are analyzed.

Several frameworks for the analysis of turbulent flows are illustrated along with examples of results obtained within such frameworks. The variability of scales and phenomena typical of river flows generate a diversity of methods for solving problems. The simplifications needed for practical applications, such as the simplifications mentioned above, which include considering stationarity and homogeneity, as well as the eventual reduction of spatial dimensions for the analysis (i.e., from 3D to 2D), are also proposed, and examples are given.

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