## Contents

1 Introduction .................................................. 1  
   1.1 A Brief History of Quantum Mechanics .......... 1  
   1.2 Revolutionary Concepts of Quantum Mechanics ... 5  
   1.3 Quantum Information .................................. 7  
   1.4 Content of the Book .................................... 10  
   1.5 Suggested Paths ........................................ 13  
   1.6 Conventions on Notation ............................... 14  
   References .................................................. 16  

Part I Fundamentals

2 Vector and Hilbert Spaces ................................. 21  
   2.1 Introduction .............................................. 21  
   2.2 Vector Spaces ............................................ 22  
   2.3 Inner-Product Vector Spaces ........................... 25  
   2.4 Definition of Hilbert Space ......................... 29  
   2.5 Linear Operators ........................................ 33  
   2.6 Eigenvalues and Eigenvectors ....................... 38  
   2.7 Outer Product. Elementary Operators ............... 40  
   2.8 Hermitian and Unitary Operators .................... 44  
   2.9 Projectors ............................................... 47  
   2.10 Spectral Decomposition Theorem (EID) .......... 54  
   2.11 The Eigendecomposition (EID) as Diagonalization . 60  
   2.12 Functional Calculus ................................... 62  
   2.13 Tensor Product .......................................... 67  
   2.14 Other Fundamentals Developed Throughout the Book . 74  
   References .................................................. 75
3 Elements of Quantum Mechanics

3.1 Introduction ................................................. 77
3.2 The Environment of Quantum Mechanics .................. 78
3.3 On the Statistical Description of a Closed Quantum System . 81
3.4 Dynamical Evolution of a Quantum System .................. 86
3.5 Quantum Measurements ....................................... 91
3.6 Measurements with Observables ............................ 98
3.7 Generalized Quantum Measurements (POVM) ............... 102
3.8 Summary of Quantum Measurements .......................... 105
3.9 Combined Measurements .................................... 106
3.10 Composite Quantum Systems .................................. 111
3.11 Nonunicity of the Density Operator Decomposition ..... 117
3.12 Revisiting the Qubit and Its Description .................. 121

References ......................................................... 129

Part II Quantum Communications

4 Introduction to Part II: Quantum Communications ............ 133
4.1 A General Scheme of a Telecommunications System ........ 135
4.2 Essential Performances of a Communication System ........ 137
4.3 Classical and Quantum Communications Systems .......... 143
4.4 Scenarios of Classical Optical Communications ............ 146
4.5 Poisson Processes ............................................. 155
4.6 Filtered Poisson Processes .................................... 158
4.7 Optical Detection: Semiclassical Model ..................... 165
4.8 Simplified Theory of Photon Counting and Implementation .. 175

References ......................................................... 181

5 Quantum Decision Theory: Analysis and Optimization ........ 183
5.1 Introduction .................................................. 183
5.2 Analysis of a Quantum Communications System ............. 186
5.3 Analysis and Optimization of Quantum Binary Systems ...... 192
5.4 Binary Optimization with Pure States ....................... 195
5.5 System Specification in Quantum Decision Theory .......... 203
5.6 State and Measurement Matrices with Pure States .......... 204
5.7 State and Measurement Matrices with Mixed States ........ 204
5.8 Formulation of Optimal Quantum Decision .................. 209
5.9 Holevo’s Theorem .............................................. 211
5.10 Numerical Methods for the Search for Optimal Operators ... 213
5.11 Kennedy’s Theorem .......................................... 216
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.12</td>
<td>The Geometry of a Constellation of States</td>
<td>221</td>
</tr>
<tr>
<td>5.13</td>
<td>The Geometrically Uniform Symmetry (GUS)</td>
<td>230</td>
</tr>
<tr>
<td>5.14</td>
<td>Optimization with Geometrically Uniform Symmetry</td>
<td>235</td>
</tr>
<tr>
<td>5.15</td>
<td>State Compression in Quantum Detection</td>
<td>238</td>
</tr>
<tr>
<td>6</td>
<td>Quantum Decision Theory: Suboptimization</td>
<td>251</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>251</td>
</tr>
<tr>
<td>6.2</td>
<td>Square Root Measurements (SRM)</td>
<td>253</td>
</tr>
<tr>
<td>6.3</td>
<td>Performance Evaluation with the SRM Decision</td>
<td>257</td>
</tr>
<tr>
<td>6.4</td>
<td>SRM with Mixed States</td>
<td>262</td>
</tr>
<tr>
<td>6.5</td>
<td>SRM with Geometrically Uniform States (GUS)</td>
<td>265</td>
</tr>
<tr>
<td>6.6</td>
<td>SRM with Mixed States Having the GUS</td>
<td>272</td>
</tr>
<tr>
<td>6.7</td>
<td>Quantum Compression with SRM</td>
<td>276</td>
</tr>
<tr>
<td>6.8</td>
<td>Quantum Chernoff Bound</td>
<td>277</td>
</tr>
<tr>
<td>7</td>
<td>Quantum Communications Systems</td>
<td>281</td>
</tr>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>281</td>
</tr>
<tr>
<td>7.2</td>
<td>Overview of Coherent States</td>
<td>282</td>
</tr>
<tr>
<td>7.3</td>
<td>Constellations of Coherent States</td>
<td>287</td>
</tr>
<tr>
<td>7.4</td>
<td>Parameters in a Constellation of Coherent States</td>
<td>292</td>
</tr>
<tr>
<td>7.5</td>
<td>Theory of Classical Optical Systems</td>
<td>296</td>
</tr>
<tr>
<td>7.6</td>
<td>Analysis of Classical Optical Binary Systems</td>
<td>304</td>
</tr>
<tr>
<td>7.7</td>
<td>Quantum Decision with Pure States</td>
<td>314</td>
</tr>
<tr>
<td>7.8</td>
<td>Quantum Binary Communications Systems</td>
<td>316</td>
</tr>
<tr>
<td>7.9</td>
<td>Quantum Systems with OOK Modulation</td>
<td>318</td>
</tr>
<tr>
<td>7.10</td>
<td>Quantum Systems with BPSK Modulation</td>
<td>320</td>
</tr>
<tr>
<td>7.11</td>
<td>Quantum Systems with QAM Modulation</td>
<td>323</td>
</tr>
<tr>
<td>7.12</td>
<td>Quantum Systems with PSK Modulation</td>
<td>331</td>
</tr>
<tr>
<td>7.13</td>
<td>Quantum Systems with PPM Modulation</td>
<td>337</td>
</tr>
<tr>
<td>7.14</td>
<td>Overview of Squeezed States</td>
<td>348</td>
</tr>
<tr>
<td>7.15</td>
<td>Quantum Communications with Squeezed States</td>
<td>354</td>
</tr>
<tr>
<td>8</td>
<td>Quantum Communications Systems with Thermal Noise</td>
<td>361</td>
</tr>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>361</td>
</tr>
<tr>
<td>8.2</td>
<td>Representation of Thermal Noise</td>
<td>363</td>
</tr>
<tr>
<td>8.3</td>
<td>Noisy Coherent States as Gaussian States</td>
<td>367</td>
</tr>
<tr>
<td>8.4</td>
<td>Discretization of Density Operators</td>
<td>369</td>
</tr>
<tr>
<td>8.5</td>
<td>Theory of Classical Optical Systems with Thermal Noise</td>
<td>373</td>
</tr>
<tr>
<td>8.6</td>
<td>Check of Gaussianity in Classical Optical Detection</td>
<td>376</td>
</tr>
<tr>
<td>8.7</td>
<td>Quantum Communications Systems with Thermal Noise</td>
<td>381</td>
</tr>
</tbody>
</table>
9  Implementation of QTLC Systems ........................................ 421
  9.1  Introduction .................................................................. 421
  9.2  Components for Quantum Communications Systems .......... 423
  9.3  Classical Optical Communications Systems .................... 431
  9.4  Binary Quantum Communications Systems ..................... 433
  9.5  Multilevel Quantum Communications Systems ................. 443
References ........................................................................... 446

Part III  Quantum Information

10  Introduction to Quantum Information .............................. 451
  10.1  Introduction .............................................................. 451
  10.2  Partial Trace and Reduced Density Operators ................ 454
  10.3  Overview of Entanglement ......................................... 457
  10.4  Purification of Mixed States ....................................... 461
References ........................................................................... 462

11  Fundamentals of Continuous Variables .............................. 463
  11.1  Introduction .............................................................. 464
  11.2  From Discrete to Continuous in Quantum Mechanics .......... 466
  11.3  The Harmonic Oscillator .......................................... 473
  11.4  Coherent States ....................................................... 479
  11.5  Abstract Formulation of Continuous Quantum Variables .... 481
  11.6  Phase Space Representation: Preliminaries .................... 484
  11.7  Phase Space Representation: Definitions for the N-Mode .... 491
  11.8  Phase Space Representations in the Single Mode ............ 499
  11.9  Examples of Continuous States in the Single Mode .......... 503
  11.10 Gaussian Transformations and Gaussian Unitaries .......... 508
  11.11 Gaussian Transformations in the N-Mode ..................... 512
  11.12 N-Mode Gaussian States .......................................... 519
  11.13 Normal Ordering of Gaussian Unitaries ........................ 522
  11.14 Gaussian Transformations in the Single Mode ............... 525
  11.15 Single-Mode Gaussian States and Their Statistics .......... 529
  11.16 More on Single-Mode Gaussian States ......................... 535
### 11 Gaussian States and Transformations in the Two-Mode

11.17 Gaussian States and Transformations in the Two-Mode ........ 540
11.18 Beam Splitter .............................................. 546
11.19 Entanglement in Two-Mode Gaussian States ................. 549
11.20 Gaussian States and Geometrically Uniform Symmetry .. 552
References .......................................................... 571

### 12 Classical and Quantum Information Theory

12 Classical and Quantum Information Theory .................... 573
12.1 Introduction .................................................. 573
12.2 Messages of Classical Information .......................... 577
12.3 Measure of Information and Classical Entropy .............. 580
12.4 Quantum Entropy ............................................. 585
12.5 Classical Data Compression (Source Coding) ............... 595
12.6 Quantum Data Compression .................................. 600
12.7 Classical Channels and Channel Encoding ................. 605
12.8 Quantum Channels and Open Systems ..................... 614
12.9 Accessible Information and Holevo Bound ................. 620
12.10 Transmission Through a Noisy Quantum Channel ...... 625
References .......................................................... 636

### 13 Applications of Quantum Information

13 Applications of Quantum Information .......................... 639
13.1 Introduction .................................................. 639
13.2 Quantum Random Number Generation ...................... 640
13.3 Introduction to Quantum Cryptography ...................... 645
13.4 Quantum Key Distribution (QKD) .......................... 646
13.5 Teleportation .................................................. 659
References .......................................................... 662

**Index** .......................................................... 665
Quantum Communications
Cariolaro, G.
2015, XXI, 673 p. 221 illus., 65 illus. in color., Hardcover
ISBN: 978-3-319-15599-9