Contents

1 Brief History of Suspension Bridges ........................................ 1
  1.1 First Suspension Bridges ............................................. 1
  1.2 Collapses Due to an External Resonance ......................... 4
  1.3 Collapses Due to Unexpected Oscillations ....................... 7
  1.4 The Tacoma Narrows Bridge Collapse ............................ 12
  1.5 Some Bridges That Did Not Collapse ............................... 14
  1.6 Some Doubts and Questions ........................................ 18
  1.7 Partial Explanations of the Tacoma Narrows Bridge Collapse... 21
    1.7.1 Structural Failure ....................................... 21
    1.7.2 External Resonance ..................................... 24
    1.7.3 Vortices ............................................... 25
    1.7.4 Flutter ............................................... 28
    1.7.5 Parametric Resonance .................................. 32
    1.7.6 Partial Conclusions: Aerodynamic Effects ................. 34
  1.8 Nonlinear Behavior of Suspension Bridges ....................... 36
  1.9 Bibliographical Notes .......................................... 40

2 One Dimensional Models ................................................. 43
  2.1 From Navier to Melan ............................................... 44
  2.2 Linear and Quasilinear Beam Equations ........................... 46
  2.3 Deflection of Cables Under Vertical Loads ...................... 49
  2.4 Suspension Bridges Modeled by Beams and Cables ............... 51
  2.5 The Melan Equation ............................................. 54
    2.5.1 How to Compute the Additional Tension of the Cables ...... 54
    2.5.2 Existence and Uniqueness Results ........................ 59
    2.5.3 Numerical Implementations with a Stable Fixed Point ...... 63
    2.5.4 Numerics with an Unstable Fixed Point
      for an Actual Bridge ......................................... 66
2.6 Self-excited Oscillations in Semilinear Beam Equations .......... 72
  2.6.1 A Model with Superlinear Springs ................................ 72
  2.6.2 Unbounded Beams and Self-excited Oscillations .......... 74
  2.6.3 Hinged Beams Subject to Nonlinear Elastic Forces .......... 78
2.7 The Birth of Aerodynamics ........................................... 85
  2.7.1 From Melan Until the Wake of Tacoma ....................... 85
  2.7.2 More Recent Models and the Sin of Mathematics ........... 89
2.8 McKenna and the Awakening of Nonlinearity ......................... 90
  2.8.1 Beam Suspended by Possibly Slackening Hangers ........... 91
  2.8.2 A Cable-Beam System with Possibly Slackening Hangers ... 95
  2.8.3 Stretching Energy in a Compressed Beam .................... 97
2.9 Bibliographical Notes .................................................. 99

3 A Fish-Bone Beam Model ................................................. 105
  3.1 A Beam Showing Torsional Oscillations .......................... 106
  3.2 Parametric Resonance in a Linearised Model .................... 107
  3.3 A Nonlinear Version ................................................ 108
    3.3.1 Well Posedness .............................................. 108
    3.3.2 Dropping the Trigonometric Functions ...................... 110
    3.3.3 Choosing the Nonlinearity .................................. 112
  3.4 Finite Dimensional Torsional Stability .......................... 114
    3.4.1 Why Can We Neglect High Torsional Modes? ............... 114
    3.4.2 Stability of the Low Modes ................................ 116
    3.4.3 The Approximated 1-Mode System .......................... 119
    3.4.4 The Approximated 2-Modes System ........................ 122
  3.5 The Flutter Energy ................................................ 125
  3.6 Which Residual Mode Captures the Energy of the Dominant Mode?
    3.6.1 Stability for Low Energy ................................... 127
    3.6.2 Numerical Computation of the Flutter Energy ............. 130
    3.6.3 More General Nonlinearities ............................... 135
    3.6.4 Mechanical Interpretation and Structural Remedies ....... 137
  3.7 The Role of Aerodynamic Forces .................................. 139
    3.7.1 Numerical Results .......................................... 139
    3.7.2 The Pattern Creating Oscillations in Suspension Bridges .. 143
  3.8 Brief History of the Hill and the Mathieu Equations .......... 145
  3.9 Bibliographical Notes ............................................... 146

4 Models with Interacting Oscillators .................................. 149
  4.1 Coupled Oscillators Modeling the Cross Section of a Bridge .... 149
  4.2 Energy Transfer and Poincaré Maps ................................ 153
  4.3 A Link Between the Poincaré Maps and the Hill Equations ...... 160
  4.4 Interactions Between Multiple Cross Sections .................... 162
  4.5 Computation of the Flutter Energy ................................ 168