Of Fermi and Paradox

Before considering the merits of the various proposed solutions to the Fermi paradox, this chapter presents some of the background. I first give a short biography of Enrico Fermi himself, focusing on just a few of his many and varied scientific accomplishments. I mention only those contributions to science that I refer to in later sections of the book. I ignore, for example, his contribution to cosmic ray physics: Fermi was the first to propose a realistic model for explaining the origin of the high-energy particles that bombard Earth from space. This work is honored by the naming of NASA’s satellite mission for investigating cosmic rays—the Fermi Gamma-ray Space Telescope. Indeed, Fermi’s scientific achievements were so numerous that the Fermi Space Telescope is only the latest in a variety of things named after him. Fermilab, in Batavia, IL, is one of the world’s leading centers for particle physics; the element with atomic number 100, which was first synthesized in 1952 in a hydrogen bomb explosion, is called fermium (Fm); the typical length scale in nuclear physics, $10^{-15}$ m, is called the fermi; 8103 Fermi is a main-belt asteroid and Fermi is a large crater on the far side of the Moon; several members of the Enrico Fermi Institute at Chicago University have won Nobel prizes. For more details of Fermi’s life, both inside and outside science, I recommend the interested reader to the biographies of Fermi listed in the References.

I then discuss the notion of paradox, and briefly look at a few examples from various fields. Paradox has played an important role in intellectual history, helping thinkers to widen their conceptual framework and sometimes forcing them to accept quite counterintuitive notions. It’s interesting to compare the Fermi paradox with these more established paradoxes.

Finally, I discuss how the Fermi paradox itself—where is everybody?—came into being. It’s worth noting that some people argue that this is neither a paradox nor is it Fermi’s. Nevertheless, we shall see that Fermi’s question can be cast into the shape of a formal paradox (if you feel the need to do so) and I explain how Fermi’s name came to be attached to a paradox that is older than many people believe.
The Physicist Enrico Fermi

It is no good to try to stop knowledge from going forward. Ignorance is never better than knowledge.

Enrico Fermi

Enrico Fermi was the most complete physicist of the last century—a world-class theoretician who carried out experimental work of the highest order. No other physicist since Fermi has switched between theory and experiment with such ease, and it’s unlikely that anyone will do so again. The field has become too large to permit such crossover.

Fermi was born in Rome on 29 September 1901, the third child of Alberto Fermi, a civil servant, and Ida DeGattis, a schoolteacher. He showed precocious ability in mathematics, and as an undergraduate student of physics at the Scuola Normale Superiore in Pisa he quickly outstripped his teachers.

His first major contribution to physics was an analysis of the behavior of certain fundamental particles that make up matter. These particles—such as protons, neutrons and electrons—are now called fermions in his honor. Fermi showed how, when matter is compressed so that identical fermions are brought close together, a repulsive force comes into play that resists further compression. This fermionic repulsion plays an important role in our understanding of phenomena as diverse as the thermal conductivity of metals and the stability of white dwarf stars.

Soon after, Fermi’s theory of beta decay (a type of radioactivity in which a massive nucleus emits an electron) cemented his international reputation. The theory demanded that a ghostly particle be emitted along with the electron, a particle he called the neutrino—“little neutral one”. Not everyone believed in the existence of this hypothetical fermion, but Fermi was proved correct. Physicists finally detected the neutrino in 1956. Although the neutrino remains rather intangible in terms of its reluctance to react with normal matter, its properties play a profound role in present-day astronomical and cosmological theories.

In 1938 Fermi was awarded the Nobel prize for physics, partly in recognition of a technique he developed to probe the atomic nucleus. His technique led him to the discovery of new radioactive elements; by bombarding the naturally occurring elements with neutrons, he produced more than 40 artificial radioisotopes. The award also recognized his discovery of how to make neutrons move slowly. This might seem a minor point but it has profound practical applications, since slow-moving neutrons are more effective than fast neutrons at inducing radioactivity. (A slow neutron spends more time in the...
neighborhood of a target nucleus, and so is more likely to interact with the nucleus. In a similar way, a well-aimed golf ball is more likely to sink into the hole if it’s moving slowly: a fast-moving putt can roll by.) This principle is used in the operation of nuclear reactors.

News of the award was tempered by the worsening political situation in Italy. Mussolini, increasingly influenced by Hitler, initiated an anti-Semitic campaign. Italy’s fascist government passed laws that were copied directly from the Nazi Nuremberg edicts. The laws didn’t directly affect Fermi or his two children, who were considered to be Aryans, but Fermi’s wife, Laura, was Jewish. They decided to leave Italy, and Fermi accepted a position in America.
Two weeks after arriving in New York, news reached Fermi that German and Austrian scientists had demonstrated nuclear fission. Einstein, after some prompting, wrote his historic letter to Roosevelt alerting the President to the probable consequences of nuclear fission. Citing work by Fermi and colleagues, Einstein warned that a nuclear chain reaction might be set up in a large mass of uranium—a reaction that could lead to the release of vast amounts of energy. Roosevelt was concerned enough to fund a program of research into the defense possibilities. Fermi was deeply involved in the program.

**Fermi Questions** Fermi’s colleagues were in awe of him for his uncanny ability to see straight to the heart of a physical problem and describe it in simple terms. They called him the Pope because he seemed infallible. Almost as impressive was the way he estimated the magnitude of an answer (often by doing complex calculations in his head). Fermi tried to inculcate this facility in his students. He would demand of them, without warning, answers to seemingly unanswerable questions. How many grains of sand are there on the world’s beaches? How far can a crow fly without stopping? How many atoms of Caesar’s last breath do you inhale with each lungful of air? Such “Fermi questions” (as they are now known) required students to draw upon their understanding of the world and their everyday experience and make rough approximations, rather than rely on bookwork or prior knowledge.

The archetypal Fermi question is one he asked his American students: “How many piano tuners are there in Chicago?” We can derive an informed estimate, as opposed to an uninformed guess, by reasoning as follows.

First, suppose that Chicago has a population of 3 million people. (I haven’t checked an almanac to see whether this is correct; but making explicit estimates in the absence of certain knowledge is the whole point of the exercise. Chicago is a big city, but not the biggest in America, so we can be confident that the estimate is unlikely to be in error by more than a factor of 2. Since we have explicitly stated our assumption we can revisit the calculation at a later date, and revise the answer in the light of improved data.) Second, assume that families, rather than individuals, own pianos and ignore those pianos belonging to institutions such as schools, universities and orchestras. Third, if we assume that a typical family contains 5 members, then our estimate is that there are 600,000 families in Chicago. We know that not every family owns a piano; our fourth assumption is that 1 family in 20 owns a piano. We thus estimate there are 30,000 pianos in Chicago. Now ask the question: how many tunings would 30,000 pianos require in 1 year? Our fifth assumption is that a typical piano will require tuning once per year—so 30,000 piano tunings take place in Chicago each year. Assumption six: a piano tuner can tune 2 pianos per day and works on 200 days in a year. An individual piano tuner therefore tunes 400 instruments in 1 year. In order to accommodate the total number of tunings required, Chicago must be home to 30,000 piano tuners. We want an estimate, not a precise figure, so finally we round this number up to an even 100.

As we shall see later, Fermi’s ability to grasp the essentials of a problem manifested itself when he posed the question: “where is everybody?”

Physicists had many questions to answer before they could build a bomb, and it was Fermi who answered many of them. On 2 December 1942, in a makeshift laboratory constructed in a squash court under the West Stands of the University of Chicago stadium, Fermi’s group successfully achieved the first self-sustaining nuclear reaction. The reactor, or pile, consisted of slugs of purified uranium—about 6 tons in all—arranged within a matrix of graphite.
The graphite slowed the neutrons, enabling them to cause further fission and maintain the chain reaction. Control rods made of cadmium, which is a strong neutron absorber, controlled the rate of the chain reaction. The pile went critical at 2:20 PM, and the first test was run for 28 minutes.

Fermi, with his unmatched knowledge of nuclear physics, played an important role in the Manhattan Project. He was there in the Alamogordo desert on 15 July 1945, just 9 miles away from ground zero at the Trinity test. He lay on the ground facing in the direction opposite the bomb. When he saw the flash from the immense explosion he got to his feet and dropped small pieces of paper from his hand. In still air the pieces of paper would have fallen to his feet, but when the shock wave arrived, a few seconds after the flash, the paper moved horizontally due to the displacement of air. In typical fashion, he measured the displacement of the paper; since he knew the distance to the source, he could immediately estimate the energy of the explosion.

After the war, Fermi returned to academic life at the University of Chicago and became interested in the nature and origin of cosmic rays. In 1954, however, he was diagnosed with stomach cancer. Emilio Segré, Fermi’s lifelong friend and colleague, visited him in hospital. Fermi was resting after an exploratory operation, and was being fed intravenously. Even at the end, according to Segré’s touching account, Fermi retained his love of observation and calculation: he measured the flux of the nutrient by counting drops and timing them with a stopwatch.

Fermi died on 29 November 1954, at the early age of 53.

Paradox

*These are old fond paradoxes, to make fools laugh i’ the alehouse.*

William Shakespeare, *Othello*, Act II, Scene 1

Our word paradox comes from two Greek words: *para* meaning “contrary to” and *doxa* meaning “opinion”. It describes a situation in which, alongside one opinion or interpretation, there’s another, mutually exclusive opinion. The word has taken on a variety of subtly different meanings, but at the core of each usage is the idea of a contradiction. Paradox is more than mere inconsistency, though. If you say “it’s raining, it’s not raining” then you’ve contradicted yourself, but paradox requires more than this. A paradox arises when you begin with a set of seemingly self-evident premises and then deduce a conclusion that undermines them. If your cast-iron argument proves it must be raining, but you look and see that it’s dry outside, then you have a paradox to resolve.
Fig. 2.2 A visual paradox. This impossible figure is a Penrose triangle. It's named after Roger Penrose, a British mathematician who devised it in the 1950s. (It was first created even earlier, in 1934, by the Swedish graphic artist Oscar Reutersvärd.) The illustration appears to show a three-dimensional triangular solid, but the triangle is impossible to construct. Each vertex of a Penrose triangle is in fact a perspective view of a right angle. Artists such as Escher and Reutersvärd delighted in presenting visual paradoxes. (Credit: Tobias R.)

A weak paradox or fallacy can often be clarified with a little thought. The contradiction usually arises because of a mistake in a chain of logic leading from premises to conclusion. For example, beginning students of algebra often construct “proofs” of obviously untrue statements such as $1 + 1 = 1$. Such “proofs” usually contain a step in which an equation is divided by zero. This is the source of the fallacy, since dividing by zero is inadmissible in arithmetic: if you divide by zero you can “prove” anything at all. In a strong paradox, however, the source of a contradiction is not immediately apparent; centuries can pass before matters are resolved. A strong paradox has the power to challenge our most cherished theories and beliefs. Indeed, as the mathematician Anatol Rapoport once remarked:8 “Paradoxes have played a dramatic part in intellectual history, often foreshadowing revolutionary developments in science, mathematics and logic. Whenever, in any discipline, we discover a problem that cannot be solved within the conceptual framework that supposedly should apply, we experience shock. The shock may compel us to discard the old framework and adopt a new one.”

Paradoxes abound in logic and mathematics and physics, and there’s a type for every taste and interest.

A Few Logical Paradoxes

An old paradox, contemplated by philosophers since the middle of the 4th century BC and still discussed, is that of the liar paradox. Its most ancient attribution is to Eubulides of Miletus, who asked: “A man says he is lying; is what he says true or false?” Whichever way one analyzes the sentence, there's
a contradiction. The same paradox appears in the New Testament. St. Paul, in his letter to Titus, the first bishop of Crete, wrote: “One of themselves, even a prophet of their own, said the Cretans are always liars.” It’s not clear whether Paul was aware of the problem in his sentence, but when self-reference is allowed paradox is almost inevitable.

One of the most important tools of reasoning we possess is the sorites. In logicians’ parlance, a sorites is a chain of linked syllogisms: the predicate of one statement becomes the subject of the following statement. The statements below form a typical example of a sorites:

- all ravens are birds;
- all birds are animals;
- all animals require water to survive.

Following the chain we must logically conclude: all ravens need water to survive.

Sorites are important because they allow us to make conclusions without covering every eventuality in an experiment. In the example above, we don’t need to deprive ravens of water to know that doing so would cause them to die of thirst. But sometimes the conclusion of a sorites can be absurd: we have a sorites paradox. For example, if we accept that adding one grain of sand to another grain of sand doesn’t make a heap of sand, and given that a single grain doesn’t itself constitute a heap, then we must conclude that no amount of sand can make a heap. And yet we see heaps of sand. The source of such paradoxes lies in the intentional vagueness of a word such as “heap”. Another paradox—Theseus’ paradox—hangs on the vagueness of the word “same”: if you restore a wooden ship by replacing each and every plank, is it the same ship? Politicians, of course, routinely take advantage of these linguistic tricks.

In addition to sorites, we all routinely employ induction—the drawing of generalizations from specific cases—when reasoning. For example, whenever we see something drop, it falls down: using induction we propose a general law, namely that when things drop they always fall down and never up. Induction is such a useful technique that anything casting doubt on it is troubling. Consider Hempel’s raven paradox. Suppose an ornithologist, after years of field observation, has observed hundreds of black ravens. The evidence is enough for her to suggest the hypothesis that “all ravens are black”. This is the standard process of scientific induction. Every time the ornithologist sees a black raven it’s a small piece of evidence in favor of her hypothesis. Now, the statement “all ravens are black” is logically equivalent to the statement “all non-black things are non-ravens”. If the ornithologist sees a piece of white chalk, then the observation is a small piece of evidence in favor of the hypothesis that “all non-black things are non-ravens”—but therefore it must be evidence for her claim that ravens are black. Why should an observation regarding chalk be evidence for
a hypothesis regarding birds? Does it mean that ornithologists can do valuable work whilst sat indoors watching television, without bothering to watch a bird in the bush?

Another paradox in logic is that of the unexpected hanging, wherein a judge tells a condemned man: “You will hang one day next week but, to spare you mental agony, the day that the sentence will be carried out will come as a surprise.” The prisoner reasons that the hangman can’t wait until Friday to carry out the judge’s order: so long a delay means everyone will know the execution takes place that day—the execution will not come as a surprise. So Friday is out. But if Friday is ruled out, Thursday is ruled out by the same logic. Ditto Wednesday, Tuesday and Monday. The prisoner, mightily relieved, reasons that the sentence can’t possibly take place. Nevertheless, he’s completely surprised when the executioner leads him to the gallows on Thursday! This argument—which also goes under the name of the “surprise examination paradox” and the “prediction paradox”—has generated a huge literature.11

A Few Scientific Paradoxes

Although it’s often fun, and occasionally useful, to ponder liars, ravens and condemned men, arguments involving logical paradoxes too frequently—for my taste at least—degenerate into a discussion over the precise meaning and usage of words. Such discussions are fine if one is a philosopher, but for my money the really fascinating paradoxes are those that can be found in science.

The twin paradox, which involves the special relativistic phenomenon of time dilation, is perhaps one of the most famous. Suppose one twin stays at home while the other twin travels to a distant star at close to the speed of light. To the stay-at-home twin, his sibling’s clock runs slow: his twin ages more slowly than he does. Although this phenomenon is contrary to common sense, it’s an experimentally verified fact. But surely relativity tells us that the traveling twin can consider himself to be at rest? From his point of view, the clock of the earthbound twin runs slow; the stay-at-home twin should be the one who ages slowly. So what happens when the traveler returns? They can’t both be right. It’s impossible for both twins to be younger than each other! The resolution of this paradox is easy: the confusion arises from a misapplication of special relativity. The two scenarios aren’t interchangeable because it’s only the traveling twin who accelerates to light speed, decelerates at the half-way point of his journey, and does it all again on the trip back. Everyone can agree that the stay-at-home twin undergoes no such acceleration. So the traveler ages more slowly than the earthbound twin; he returns to find his brother aged, or even dead. An extraterrestrial visitor to Earth would observe the same phenomenon when it returned to its home planet: its stay-at-home siblings (if aliens have siblings)
would be older or long-since dead. This behavior is certainly contrary to our experience, but it’s not a paradox—rather, a sad fact of interstellar travel.\textsuperscript{12}

The so-called firewall paradox is of much more recent vintage than the twin paradox. It was first proposed in 2012,\textsuperscript{13} and since then a storm of papers have attempted to resolve the underlying riddle. As of the time of writing, no one has managed to douse the firewall; it remains a troubling issue for theoretical physics. The paradox arises because of an apparent contradiction between the predictions made by three fundamental theories of physics: quantum theory, general relativity and complementarity.

Quantum theory is our best theory of the physical processes that happen in nature. It’s a probabilistic theory, which means that it doesn’t predict what will definitely happen; rather, it gives the \textit{probability} that some particular event will happen. Quantum theory thus only makes sense if the probabilities of all the different outcomes to an event add up to 1. If you add up the probabilities for all possible outcomes and find that the result is 0.8 or 1.3—or any value except 1—then the result is nonsensical. It follows that information in quantum theory can’t be lost and it can’t be cloned: if information somehow disappeared or could somehow be copied then probabilities wouldn’t add up to 1 and the result would be nonsense.

General relativity, which is our best theory of gravity, is a classical rather than a quantum theory. In other words it gives a definite prediction for the outcome of an event rather than a range of probabilities for different possible outcomes. General relativity describes gravity in terms of the warping of spacetime, and one of its predictions is that when the warping of spacetime becomes intense enough a black hole can form. A black hole is a region of space where not even light itself travels fast enough to escape the grip of gravity. Surrounding a black hole is an event horizon, a “surface of no return”. If you are outside the event horizon then it’s always possible, if only in principle, to leave the vicinity of the black hole; fall over the event horizon, however, and any attempt to leave the black hole will inevitably end in failure. It’s important to note that according to general relativity you wouldn’t notice anything special as you passed the event horizon; there’s no sign marking the boundary in space beyond which lies a black hole. The usual analogy is with a rowing boat on a river with an increasingly fast current that culminates in a weir. The river contains a point of no return, beyond which the muscle power of any rower will fail to overcome the current. If the boat passes the point of no return then its fate is sealed: it will be carried over the weir. But nothing in the river marks that point of no return, and the boat can drift quite peacefully past that point without noticing anything has changed. It’s the same with the event horizon surrounding a black hole.

In the mid-1970s, Stephen Hawking introduced the black hole information paradox to physics. Hawking showed that black holes do in fact radiate: quantum effects close to the surface of the event horizon mean that particles can
leave the vicinity of the horizon. Black holes emit so-called Hawking radiation, and this radiation carries information and energy with it. This effect causes the black hole to lose energy, which in turn means that it shrinks. Eventually, the black hole evaporates. The question is: what happens to the information that was inside the black hole? If the information was carried away by Hawking radiation then the information would have had to have been cloned: the information couldn’t have escaped from inside the event horizon. But having two copies of information violates quantum theory because it would mean probabilities don’t add up to one. So perhaps the information disappears when the black hole evaporates? But disappearing information violates quantum theory because it would mean probabilities don’t add up to one. We have a paradox: quantum theory and general relativity appear to give conflicting accounts about what happens to any information that might fall into a black hole.

In the early 1990s, Leonard Susskind and co-workers proposed something called complementarity as a resolution of the black hole information paradox. Susskind’s idea was that in a sense the problem is one of perspective: observers inside and outside the event horizon see different things. An observer outside the black hole sees information gather at the event horizon and then eventually flee the black hole in Hawking radiation. An observer inside the black hole sees information as being inside the event horizon. Since the two observers can’t communicate, the paradox is avoided. Susskind’s proposal in a sense allows the information to be both inside and outside the event horizon in a way that doesn’t violate the requirements of quantum theory. His proposal was given a boost in 1997, when Juan Maldacena proposed an idea called AdS/CFT correspondence. The idea says that string theory (which automatically contains gravity) is equivalent to a quantum theory without gravity in a space of fewer dimensions. Maldacena’s paper has been hugely influential, because it allows physicists to attack problems that would otherwise be too difficult: if a problem is intractable in one regime simply switch to another regime where it might be tractable, do the work there, then switch back to the original regime. Crazy as it might sound, the AdS/CFT correspondence states that the three-dimensional interior of a black hole with gravity is equivalent to a quantum theory without gravity that sits just above the two-dimensional surface of the horizon. A lot of theoretical work based on this correspondence seemed to back up the complementarity proposal. It seemed that information isn’t lost, quantum theory was saved and the information paradox was put to bed.

In 2012, however, four physicists (Ahmed Almheiri, Donald Marolf, Joseph Polchinski and James Sully, collectively known as AMPS) discovered something unsettling when they tried to describe the process of black hole evaporation in terms of complementarity. According to their analysis, when a black hole is about halfway through the evaporation process it has lost so much
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