Preface

The purpose of this book is to give a gentle introduction to noncommutative rings and algebras that requires fewer prerequisites than most other books on the subject. It is based on a series of lectures given to masters students at the University of Ljubljana and is intended to serve as a first reading on noncommutative algebra for beginning graduate and advanced undergraduate students. The first two chapters, in which some results of historic importance are derived by rudimentary tools, can have a wider audience. For example, they can be used in a course for future high school teachers, or for an undergraduate seminar. Mathematicians working in areas that only have some interactions with noncommutative algebra may also benefit from this book in which important, classical themes are presented in a simple manner, avoiding excessive generality.

The necessary background needed to follow this text is a standard knowledge of linear algebra and a basic knowledge about groups, rings, and fields. To make precise what we mean by this, the book begins with a survey of prerequisites. This is followed by the first chapter which considers finite dimensional division algebras. Its most prominent results are Frobenius’ theorem on real division algebras and Wedderburn’s theorem on finite division rings. The second chapter is devoted to the structure of finite dimensional algebras, featuring the classical Wedderburn’s theory. After the first two warm-up chapters, which mostly deal with results known for a hundred years, the next two introduce and study more abstract notions: modules, vector spaces over division rings, and tensor products. The Double Centralizer Theorem and the Skolem-Noether Theorem are included therein. The fifth chapter considers the structure theory of rings. The main themes are primitive rings, the Jacobson Density Theorem, and the Jacobson radical. If the first five chapters survey the “greatest hits” of noncommutative algebra, the last two are slightly more specialized, reflecting my personal taste and interests. They treat polynomial identities and related notions, such as free algebras and rings of quotients.

The order of the topics in the book is quite close to the chronological order of their development. This actually occurred unintentionally, probably as a result of my attempt to follow the principle that an advanced concept should be introduced
only when truly needed, and not at a higher level of generality than necessary. Not because I do not appreciate abstract concepts; after all, they are what makes mathematics beautiful. But it is because they can be better understood and valued when given some evidence that they are indispensable. Another principle I have followed, especially in the early chapters, is to choose proofs that appear to be the simplest. This may not always be the same as the shortest, but rather proofs that seem very natural and easy to memorize. It has been my desire to write the book in such a way that readers would not get an impression that one needs supernatural abilities to create mathematics, but that even they themselves may be able, with some luck and courage, to discover a little piece of it.

One challenge in the exposition of the book was to find new proofs of classical theorems that would fulfill the simplicity criteria described above. This has been in fact my little passion over the last years, which has eventually yielded some results. The proofs of the following theorems are different from those in the standard sources: Frobenius’ theorem (Sect. 1.1), the Skolem-Noether Theorem (Sect. 1.6), Wedderburn’s structure theorems (Sect. 2.9), Martindale’s theorem on prime GPI-rings (Sect. 7.7), Posner’s theorem on prime PI-rings (Sect. 7.9), and the Formanek-Razmyslov Theorem on central polynomials (Sect. 7.10). How much originality is there in these proofs? Frankly, I do not know. I have not found such proofs in the literature, and have published them in mathematical journals through a series of papers. On the other hand, so many mathematicians have known these theorems, especially the first three listed, for so many years that one hardly imagines that something entirely new about them can still be found. It is not so rare that some mathematical ideas are discovered, forgotten, and rediscovered. Anyway, I hope that these proofs provide interesting alternatives to those from other sources. Two remarks must be added. First, the proof of Frobenius’ theorem was obtained in a joint work with Peter Šemrl and Špela Špenko. Second, shortly after the publication of my paper on alternative proofs of Wedderburn’s structure theorems Edmund Puczylowski informed me that he had used them in his class, but with a modification—which I immediately liked. I am now happy to use this modified version in the book.

Each chapter ends with exercises of varying difficulty levels. They are sorted by topics, not by difficulty. Some of the harder ones are accompanied with hints, but others are not, in order to give the student the opportunity to fully enjoy the pleasure of discovering the solution. A fair amount of exercises are original, but there are also some that appear, either as exercises or as theorems with proofs, in almost every book on the subject. I tried to avoid long, theoretical exercises, and gave preference to those that I had found entertaining and appealing.

The list of references at the end of the book is very short. It includes only books which are explicitly referred to on relatively rare occasions when details are not provided (say, when closing the discussion on a topic with additional information, or when omitting tedious details in some example). Most of these books are textbooks on noncommutative rings and algebras, but written at a more advanced level than this one. An exception is perhaps J. Beachy’s Introductory Lectures on Rings and Modules [Bea99] whose emphasis, however, differs from ours. At the
time of writing I was frequently consulting many of the listed books, but mainly T. Y. Lam’s *A First Course in Noncommutative Rings* [Lam01] (together with the accompanying problem book [Lam95]) and L. H. Rowen’s *Graduate Algebra: Noncommutative View* [Row08]. I warmly recommend these two texts as further reading to students who want to dig deeper into the field of noncommutative algebra.

English is not my mother tongue and I apologize to the reader if this is sometimes too obvious. “Translating” the English stuck in my mind into the English as it should be has been a big and often frustrating challenge for me during the writing process. The bright side of this is that it gives me the opportunity to thank my sons Jure and Martin for their continuous help in this matter.

The most difficult step in writing a book is forcing yourself to write the first sentence. I am thankful to my friend and colleague Peter Šemrl for giving me the necessary encouragement to make it.

Rough drafts of most chapters were first read by Špela Špenko. Her criticism has been very helpful (as always). Aljaž Zalar has solved most of the exercises and made me think of modifying some. Igor Klep provided many helpful suggestions concerning Chaps. 1 and 6. The book has been read cover to cover by Nik Stopar and Janez Šter. Their comments have contributed to the improvement of the text and saved me from some errors. My sincere thanks go to all of them, as well as to the Springer staff for their professional assistance.

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