2.1 Bonds, Stocks and Commodities

Bonds
In Chapter 1, bonds have been introduced as an important class of financial assets which is structurally similar to loans. The authorized issuer promises in the bond contract to make future payments according to a fixed schedule, up to some final time $T$ (the term or maturity of the bond). The promised payments typically consist of the principal (or: face value) of the bond (e.g. 10,000 EUR) at time $T$ and a regular (for example, annual, semi-annual or quarterly) coupon (e.g. 500 EUR at the end of every year). If no coupon is paid, there is only one payment at maturity (typically after one year or less) and the bond is called zero-coupon bond. Coupon payments can be an initially fixed amount, e.g. 5% p.a. of the principal. Alternatively, the size of the coupon can be linked to some reference interest rate, e.g. Libor+1% (see Section 1.4). If the principal is paid in one lump sum at maturity, the bond is called bullet. Otherwise one speaks of an amortizing bond.

Note that the issuer will often hold an auction when initially selling the bond to investors. The initial price of the bond is determined by the bids of the investors, and can be different from the face value. Given a face value of 100, if investors offer more than 100, the bond is said to sell at a premium to par. Conversely, if investors offer less than 100, the bond sells at a discount to par. Once the bond is sold to the initial investors in the primary market, these investors might decide to sell the bond to other parties in the secondary market. Bonds are debt securities and can easily be traded privately (for example, through bond funds, insurance companies or banks), or exchanges might provide a platform to match buyers and sellers. Note that a bond investor will record the bond as an asset on its balance sheet, while the issuer will report it as a liability (i.e. as an obligation to pay money in the future).

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1Due to their fixed payment schedule, bonds are also referred to as fixed income products.
Stocks

A stock (or: share) represents capital paid into a company in return for ownership, either by the initial founders or at a later stage. A stock is a security that gives its holder a number of rights, including

- the right to receive dividends;
- the right to participate, speak and vote at General Meetings;\(^2\)
- the right to receive new shares. As additional share capital is raised, this will typically be first offered to current shareholders so that their voting power is not necessarily diluted;
- the right to participate in the distribution of liquidation proceeds once all other liabilities have been repaid in full.

Note that stocks can also be held and traded privately, they are not necessarily listed at stock exchanges. Listed companies might have a large free float, i.e. a large portion of their stocks is owned by many different equity investors, which provides sufficient liquidity for almost continuous trading. Many regulators require larger holdings of shares of a company to be (publicly) disclosed (e.g. UK: once the holding exceeds 3% of the number of outstanding shares).\(^3\),\(^4\)

Listed companies are required to publish detailed information in the form of quarterly and annual reports. Information rules can be imposed by the regulator or the respective stock exchange, and might differ from market to market.

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\(^2\)A stock company is required by law to hold Annual General Meetings where past and future activities are discussed, fiscal information is reviewed and the Board of Directors is elected.

\(^3\)Larger strategic holdings by long-term investors are not counted into the free float, together with government holdings or holdings of founding investors.

\(^4\)Stock prices of otherwise comparable companies with only a small free float can be more volatile. Some hedge funds had to experience this in 2008, as they lost more than 20bn GBP when closing short positions on Volkswagen stocks. Porsche had just announced that it had acquired as much as 74% of Volkswagen stocks. Only a relatively small portion of stocks was still free-floating, so that prices sky-rocketed within hours due to the sudden demand from hedge funds and the limited supply.
Stock Indices

To describe the performance of an entire stock market, for example a selection of companies listed at the Frankfurt stock exchange, stock indices are computed and published and can be tracked over time. A stock index is a linear combination of a set of stock prices and is published by the stock exchange itself (e.g. DAX (Frankfurt), DJIA (New York), Nikkei (Tokyo), SMI (Zurich)) or by information providers (e.g. S&P 500 (500 large cap stocks traded in the US), the Dow Jones Industrial Average (short: DJIA, 30 large US based companies that are publicly traded)).

Suppose an index contains \( n \) stocks with stock prices \( s_1, s_2, ..., s_n \) and numbers of outstanding shares \( nos_1, nos_2, ..., nos_n \). The market capital \( mc_i \) of stock \( i \) is simply its current stock price times the number of its outstanding shares, i.e. \( mc_i = s_i \cdot nos_i \).

Indices can then be calculated as price-weighted indices or market-value-weighted indices. A price-weighted index \( I_p \) is calculated as

\[
I_p = \frac{\sum_{i=1}^{n} s_i}{\text{number of stocks (adjusted for splits)}},
\]

and it will become clear from the example below how the number of stocks (adjusted for splits) is computed. A market-value-weighted index \( I_m \), on the other hand, is calculated as

\[
I_m = c \cdot \sum_{i=1}^{n} mc_i = c \cdot \sum_{i=1}^{n} s_i \cdot nos_i
\]

for some constant \( c > 0 \). Clearly, a market-value-weighted index can move with only a small number of large companies that have large market capital, while small market-capital companies have relatively more weight in a price-weighted index.

Now assume a company decides to split its stocks so that current owners receive \( k \) new stocks for every stock they own. If a stock trades at 33 GBP before the split, the new stocks just after a 1:3 split will trade at 11 GBP and each investor will hold three times as many shares as before. Stock splits have no effect on market-value weighted indices since \( s_i \cdot nos_i = \frac{s_i}{k} \cdot (nos_i \cdot k) \). To understand the effect of stock splits on price-weighted indices, consider the following example.

Example (Downward bias of price-weighted indices)

Consider a price-weighted index on a set of two stocks A and B. At time \( t_0 \), stock A trades at 100 EUR per share and stock B at 40 EUR. At some later time \( t_1 \), stock A rises to 200 EUR and stock B to 50 EUR. We calculate \( I_0 = \frac{100+40}{2} = 70 \) and \( I_1 = \frac{200+50}{2} = 125 \). Company A decides that its stock trades too high and splits it 1:2. After the split, at time \( t_1^+ \), the number of stocks has to be adjusted from 2 to number of stocks \( t_1^+ \), so that the index does not change. Hence, one solves

\[
\frac{200/2 + 50}{\text{number of stocks}_{t_1^+}} = \frac{200 + 50}{2}
\]
to find number of stocks\(_{\text{adj}}(t_1^+) = 1.2\). At some later time \(t_2\), A and B trade at 118 EUR and 50 EUR, respectively. The index will now be \(I_2 = \frac{118 + 50}{1.2} = 140\). Note that A performed relatively better than B over the period \([t_1, t_2]\). Without the stock split, the index would have been \(I_2^{\text{no split}} = \frac{236 + 50}{2} = 143\). Due to the split, stock A has lost some influence on the index. This effect is known as downward bias of price-weighted indices, because successful companies are more likely to perform stock splits when their stock price keeps rising.

Without going into further detail, keep in mind that different ways of computing indices measure market performance differently. Also, some indices are published both as price performance indices and total return indices, depending on whether dividend payments are included. Finally note that indices have become a fundamental tool of well-developed financial markets, as they allow to assess the performance of single assets relative to an entire market, to evaluate relationships between financial or economic variables and market performance, to construct index portfolios tracking the overall market, and to hedge against adverse (sub-)market movements through index-based derivatives.

### Currencies (FX)

Currency or foreign exchange (short: FX) markets provide a platform for trading currencies. Currencies are traded directly between two parties over-the-counter (short: OTC), without going through an exchange, and most trades are between banks. A particular trade consists of a currency pair, such as EUR/USD, USD/JPY, AUD/USD, or USD/CHF. A market maker could quote EUR/USD 1.2938/1.2940. EUR would be the base currency, as the quotes refer to 1 EUR, and USD the quoted currency.\(^5\) The quote is given as bid/ask, i.e. the market maker would buy 1 EUR for 1.2938 USD, and sell 1 EUR for 1.2940. The difference between the two quotes is called bid-ask spread. Currencies are typically traded in contract sizes (or: lot sizes) of 100,000 units of the base currency, but smaller sizes are also offered to retail clients. The FX market is one of the largest markets if measured by transaction volume. The average daily turnover in April 2010 was 4,000bn, which marked a 20% increase over the April 2007 figure (cf. BIS [74]). FX rates can be very volatile and Figure 2.26 depicts the development of the EUR-USD exchange rate from 1999-2012.

### Commodities

Commodities, such as oil (different types), gas, coal, electricity, base metals, precious metals, agricultural goods (soy, wheat, corn, pork bellies) or soft commodities (coffee, cocoa, sugar, cotton, orange juice), can be traded in the spot market or the forward/future market. Upon trades in the spot market, the buyer receives control over the traded good immediately or at the latest within a short settlement period.

\(^5\)Which currency in a traded pair is quoted as base currency is mostly based on historical convention.

\(^6\)Source: www.bundesbank.de
The bulk of the trades are however executed in the forward market. For example, when entering a contract in the forward market, one counterparty might accept the obligation of delivering 10 megawatt-hours of electricity per hour throughout some future month. The other counterparty then has the commitment to buy this quantity of electricity at the scheduled times at a price fixed today. We will further discuss this kind of contracts in Section 2.3.

### 2.2 Derivatives

Financial instruments whose value depends on the price of some other underlying product are called derivative instruments (short: derivatives). Derivatives that give the right (but not the obligation) to engage in a financial transaction at a later point in time are called options. An example of an option would be the right to buy or sell an asset at some later time $T$ at a price fixed today. The analysis of such contingent claims is one of the main fields of modern financial mathematics.

Derivatives can be standardized contracts that are traded at stock exchanges, or they can come in the form of products tailored specifically to the requirements of the counterparties. Such non-standard contracts are typically traded over-the-counter (OTC).

Why are derivatives traded and who would have particular interest in entering into derivative contracts? Two possible motivations for engaging in the derivatives market are listed below:

- **Hedging:** Consider the following example. An exporting company, which produces a machine in Europe, has agreed to sell this machine upon completion to a client in the US at a fixed USD amount. Assume that the production costs of this machine will mainly incur in EUR. The company is therefore exposed

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7Note that the underlying of a derivative contract can again be a derivative with respect to another underlying, and so on.
to currency exchange rate risk between the time of production and the time of the sale. An unfavorable move of the EUR/USD rate (i.e. that the USD loses value compared to the EUR) will lower the company’s profit. The company can now partly or fully mitigate this risk by entering into an FX forward contract. This contract fixes the future exchange rate at a certain level. Mitigating risk by taking on a portfolio of one or more financial instruments is called hedging. In particular, note that the exchange rate risk is now borne by the counterparty in the FX forward contract (which will often be a bank) rather than by the company or the buyer of the machine.

- **Taking uncovered positions:** Market participants can also take a position in a derivative without being in some way exposed to the underlying risk. This would be called taking an uncovered position, and it can lead to a profit if a particular market view proves true. For example, one could take the position of the counterparty in the above FX forward contract thinking that the USD will gain value against the EUR. If the USD then actually appreciates versus the EUR, this position will bring a profit. Taking positions in derivative products typically allows for more specific and efficient trading strategies than those realizable by holding positions in only the underlyings themselves (cf. Section 2.5).

### 2.3 Forwards and Futures

In the spot market, goods and payments are exchanged (e.g. domestic against foreign currency, cash against stocks, cash against copper etc.) immediately or at the latest within a short settlement period. Conversely, it can be agreed to execute the exchange at some later time. If the later exchange is unconditional, this contract type is called forward contract. Concretely, a forward contract defines the obligation to trade a good (e.g. a stock) at some time $T$ at an agreed price $F$. The buyer of the underlying is said to have a long position in the forward, and the seller has a short position. The transaction (the payment of the forward price and the delivery of the good) will be executed at time $T$. If the price $S_T$ of the underlying at time $T$ is larger than $F$, then the contract has the value $S_T - F > 0$ to the buyer. Conversely, the seller has to sell below market, and therefore takes a loss of $F - S_T$. The pay-offs

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8In our above example, the hedging portfolio consists of one FX forward contract.

9Note that we often take views when making financial decisions. For example, when part-financing the purchase of a house through a bank loan, the borrower might be able to choose between fixed or floating interest rates, or to fix an upper interest rate limit (also: cap) in the case of floating interest rates. It also used to be popular to finance real estate by loans in foreign currencies with lower borrowing rates, for instance, financing a German house with a CHF loan when interest rates in CHF were lower than in EUR. During the economic downturn starting in 2007, however, the CHF greatly appreciated in value against the EUR, so that CHF-denominated liabilities required a significantly higher EUR amount to be repaid. Even when choosing a mobile phone contract, one will usually decide on a particular contract duration/fee combination and hence take a view on phone contract terms in e.g. 12 months from now.
of the long and short forward contract are depicted in Figure 2.3. Note that only the short position faces a potentially unbounded loss.

Forwards are not only traded on underlying stocks, but also on interest rate products, other financial instruments, and commodities. The standardized version (in terms of the quality of the underlying, the maturity, the contract size, etc.) of the OTC-traded forwards are called futures. Futures are traded at futures exchanges. The standardized nature of futures makes it easier to take a counter-position to close a certain position (e.g. closing a long position by adding a short position – netting off the two pay-offs in Figure 2.3 gives then zero) and ensures increased trading liquidity. Futures exchanges include the Chicago Mercantile Exchange (www.cmegroup.com), the Intercontinental Exchange Inc. (www.theice.com) and the European Energy Exchange in Leipzig (www.eex.com).

Finally note that, in practice, instead of physical settlement (i.e. the underlying will be physically delivered against the payment of the futures price at maturity), most future contracts will be cash settled (i.e. one party will receive a payment corresponding to the value of the contract at the time of closing the position). The actual financial settlement of future contracts will be done through a clearing house as central counterparty.\(^\text{10}\) As future contracts can have a maturity of up to several years, the price of the underlying in the spot markets (and hence the value of the of the futures contract) can fluctuate significantly up to maturity of the future contract. Pricing of futures and lowering the risk of the futures counterparty not fulfilling its obligations under the contract will be further discussed in Section 3.2.

\(^{10}\)Currently (2012) LCH.Clearnet (www.lchclearnet.com) is the largest clearing house for derivatives.
2.4 Swaps

Swaps are contracts between two counterparties to exchange two cash flow streams. Consider the following example of a fixed-for-floating interest rate swap.

Example (10-year vanilla interest rate swap)

Effective/Termination date: 25 April 2012/25 April 2022
Notional amount: 8,000,000 EUR
Party A pays and party B receives: quarterly Euribor3M, fixing in advance (ACT/360)
Party B pays and party A receives: 2.320% p.a., paid annually, (30/360).

The party in an interest rate swap which pays the fixed rate is called fixed rate payer. In the above example, counterparty A is the fixed rate receiver. Cash flows under the swap (from A to B, and vice versa) are calculated by applying the respective interest rates to the notional amount, which is similar to the principal of a bond. However, the notional itself is actually never exchanged between the parties. Note that arbitrary reference interest rates can be used when defining a swap, however, for Euribor/Libor common rates include 1M, 3M, 6M or 12M. The two different cash flow streams in a swap are referred to as legs. The floating Euribor3M cash flow in the above example would be called floating leg, the cash flow linked to the fixed interest rate fixed leg. Even for more complex swap products, one leg will typically have a plain vanilla structure as above, while the structure of the other leg may be more complex. From a certain degree of complexity upwards, the contracts are called structured swaps and will be further discussed in Chapter 13.

Swaps are typically tailored to the needs of at least one of the counterparties and hence traded OTC. It has become an industry standard to document a swap contract based on a swap master agreement as developed by the International Swaps and Derivatives Association. Using standard documentation and standard contract terms considerably lowers documentation risk and legal risk, and allows to compare different contracts more easily.

The value of a swap (from the viewpoint of the respective counterparty, A or B) typically changes over its life as market conditions (e.g. interest rate levels) change. If the swap has value to e.g. party A, A bears the risk that party B will not be able or willing to entirely fulfil the contract. Hence, A might contractually require B to post some sort of collateral (e.g. cash or government bonds) to cover this risk. Initially, the fixed rates in the case of vanilla interest rate swaps are mostly set such that the swap has zero value at the beginning (and this fixed rate is referred to as swap rate). If the swap in the above example had had zero value on the 25th of April 2012, the 10-year EUR-swap rate would have been 2.320% then. Note that plain vanilla swaps are very liquid instruments, which is partly due to the standard

11ISDA, www.isda.org
Fig. 2.4 Cash flows on a loan interest payment day for the example below.

- **Tenant**: Rent = 600,000 EUR
- **Investor A**: Libor_{12M} + 2% = 7%
- **Bank B**: Libor_{12M} = 5%
- **Bank C (Swap)**: 4% payable by A

The definitions of the ISDA documentation and publicly available benchmark quotes (for example, ISDAFIX). In general, swap contracts can also have non-zero initial value, so that one counterparty would make an initial payment to the other counterparty. Similarly, one can choose a structure where the notional increases or decreases over time (accretive principal swap or amortizing swap, respectively), such that swap contracts can be tailored for managing interest rate risk arising from specific loans or bonds. We close this section with an example of how swaps can be applied to the hedging of interest rate risk.

**Example (Interest rate hedging)**

Suppose A is a real estate investor and buys a building for 12 mn EUR that produces 600,000 EUR in net rental income every year. A only has 2 mn EUR in cash and borrows the remaining 10 mn EUR from bank B for a term of 7 years and at an interest rate of Libor_{12M} + 2%. As the rental income from the tenant is fixed in the lease contract, there is the risk that Libor_{12M} rises very high, so that the interest payment to B cannot be covered from the net rental income any longer. To mitigate this risk, bank B asks A to enter into a fixed-for-floating interest rate swap contract. Another bank C offers to pay Libor_{12M} against a fixed rate of 4% paid by A (assume yearly payments). The notional is set at 10 mn EUR and the termination date is in 7 years from now. Figure 2.4 shows the cash flows on a loan interest payment day if Libor_{12M} = 5% on some fixing day. Note that A can only cover the interest due to the extra payment from the swap counterparty. Conversely, if LIBOR_{12M} was below 4% on a fixing day, A would have to pay 10 million × (4% − LIBOR_{12M}) to the swap counterparty C (in which case having a swap in place would be a disadvantage for A). A has effectively locked in its interest plus swap costs at 4%.

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2.5 Options

In the financial context, an *option* is the right, but not the obligation, to purchase or sell some underlying asset (e.g. a stock) at some time $T \geq 0$ at a pre-defined price $K$. The price $K$ is called *strike price* (or simply: *strike*) and $T$ is called *expiration date* (or: *expiry*). One distinguishes between *call options*, which give the option buyer the right to buy, and *put options*, which are rights to sell (to 'put an
The buyer of an option is said to have a long position in the option, while the seller has a short position.

The pay-off of an option is its value at the time of its exercise. In the case of a call option with strike $K$ on an underlying stock with price $S_T$ at expiry $T$, the pay-off $C_T$ is given by $S_T - K$ if $S_T > K$, and 0 if $S_T \leq K$. In the latter case the stock can be purchased at a price lower than $K$ in the market, and hence the option will not be exercised. Altogether, one can write

$$C_T = \max(S_T - K, 0) = (S_T - K)^+.$$  

Similarly, the pay-off $P_T$ of a put option is given by $P_T = (K - S_T)^+$ (see Figure 2.5).

**Example (Leverage effect of options)**

Let $S_0 = 100$ EUR be the price of a stock today, and let some call option on the stock have strike $K = 120$ EUR, expiry $T$ and initial price $C_0 = 5$ EUR. How can one profit, if the stock price will rise significantly until $T$?

(a) Buy the stock today at $S_0 = 100$. If it turns out that $S_T = 130$ EUR, the stock holder will have made a 30% profit on the investment over the period $[0, T]$.

(b) Alternatively, you could buy the call option today. If $S_T = 130$ EUR, the option will be exercised and the stock can be attained at time $T$ at 120 EUR. If the stock is then immediately sold in the market, this would give a profit of $\frac{130 - 120}{120} = 100\%$ on the investment.\(^{13}\)

The increased percentage profitability of buying the option compared to buying the underlying stock is called leverage effect. Note, however, that strategy (b) also bears the risk of receiving zero pay-off (if $S_T < 120$), so that the entire investment would be lost in that case. Similarly, one can profit from falling stock prices in a leveraged structure by buying put options.

So far we have only considered the possibility of the options being exercised on one specific date, the expiry date. Such options are called European options. Other types of options are also offered in the market. For example, American options can

\(^{12}\)Calls were first traded as standardized contracts at the CBOE (Chicago Board Options Exchange) in 1973, and puts followed in 1977. Today options are traded at more than 50 exchanges worldwide. The most important European options exchanges include EUREX (www.eurexchange.com) and LIFFE (www.liffe-commodities.com).

\(^{13}\)In practice, the option holder will typically receive a cash settlement of $130 - 120 = 10$ EUR, instead of receiving the stock physically and paying 120 EUR.
be exercised at any point in time up to expiry, or *Bermudan options* can be exercised at pre-defined discrete times up to expiry. Note that options can deviate from the *plain vanilla* structure as explained here. Such more complex options are referred to as *exotic options*, and are traded OTC. Examples of exotic options include:

- **Asian options**: the stock can be sold at expiry at the average stock price up to expiry (or, in a slightly different structure, the strike is fixed and the pay-off is given as the difference between the average stock price and the strike if this difference is greater than 0, and 0 otherwise). The price averaging dampens the effect of highs and lows in the price development of the underlying.
- **Barrier options**: in this case, the pay-off of this otherwise European option depends on whether the stock price crosses a certain barrier up to expiry. For the so-called *knock-out option*, the option is canceled (i.e. the pay-off becomes 0) as soon as the defined barrier is crossed, for the *knock-in* version, the European pay-off is only made if the barrier has been crossed.

• **Compound Options**: are options on options.
• **Digital Options**: have the constant pay-off 1, in case the stock price $S_T$ exceeds the strike $K$ at expiry, and 0 otherwise (in the case of a call).

This list could be arbitrarily extended, in particular for the remaining 22 letters of the alphabet.

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### 2.6 Key Takeaways, References and Exercises

#### Key Takeaways

After working through this chapter you should understand and be able to explain the following terms and concepts:

- Bond, bond issuer, zero-coupon bonds, bullet, principal/face value, premium/discount to par
- The rights of a stock holder
- Market-value-weighted vs. price-weighted stock indices, the downward bias
- In the context of FX, bid/ask quotes, bid/ask spread, base currency
- The difference between forwards and futures
- Swap contracts
- European, American, Asian, Bermudan and Barrier options, and the leverage effect of options

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14 Barrier options are amongst the most liquid OTC options and are an important building block of many structured products (cf. Chapter 13).
References


Exercises

1. What is the number of outstanding shares (NOS) of the Swiss company Asea Brown Boveri (ABB)? At what stock exchanges are ABB stocks listed? Plot the price development of ABB stocks over the last 5 years.
2. What stocks does the Dow Jones Industrial Average (DJIA) consist of? What is the composition of the DAX? How is the ATX calculated?
3. Check and list the contract specifications of various PHelix futures as traded at the European Energy Exchange.
4. What are the current prices of European options on the S&P500 index as listed by the CBOE?
5. (a) Explain the difference between holding a long position in a forward contract with a forward price of 50 EUR, or a long position in a call option with strike 50 EUR.
   (b) A trader expects a stock price to rise and would like to profit in case his view proves true. The current stock price is 29 EUR and a European call option (\( T = 3 \) months, \( K = 30 \) EUR) prices at 2.90 EUR. The trader can invest a total of 5,800 EUR. Identify two strategies – investing in the stock, or taking a long position in the call options. Specify the absolute and relative (percentage) profit/loss of the two strategies, depending on the stock price in 3 months from now.
6. A company has information that it will receive a certain amount in foreign currency in 4 months from now. How can you hedge this transaction using (i) a forward contract, or (ii) an option contract. What will the structural difference between (i) and (ii) be?
7. Search the internet to find out what types of Asian options are commonly used.
8. (a) Describe the pay-off of the following portfolio: a long position in a forward contract on a stock and a long position in a European put option, both with expiry \( T \). The strike \( K \) of the option shall equal the fair forward price of the stock at time 0.
   (b) Is the following statement true? Explain your answer. ‘A long position in a forward contract is equivalent to a long position in a European call option and a short position in a European put option.’