Chapter 1.2
Quantitative Emergence

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Abstract Emergence can be defined as the formation of order from disorder based on self-organisation. Humans—by looking at a self-organising system—can decide intuitively whether emergence was taking place or not. To build self-organising technical systems we need to automate the recognition of emergent behaviour. In this paper we try to give a quantitative and practically usable definition of emergence. The presented theoretical approach is applied to an experimental environment, which shows emergent behaviour. An Observer/Controller architecture with emergence detectors is introduced. The proposed definition of emergence is discussed in comparison with Shannon’s information theoretical approach.

Keywords Emergence · Self-organisation · Order · Entropy · Observer/controller architecture

1 Introduction

Organic Computing (OC) has become a major research activity in Germany and worldwide [6]. Its goal is the technical utilisation of emergence and self-organisation as observed in natural systems. Emergent and self-organising behaviour has been observed in nature, demonstrated in a variety of computer simulated systems in artificial life research, and it has occurred also in highly complex technical systems (like the Internet) where it has led to unexpected global functionality. Despite the importance of a rigorous description of these phenomena, the quantitative analysis of technical self-organising systems is still a rather unexplored area.

Emergence and self-organisation have been discussed by a variety of authors for many years. The most commonly accepted definition is that the whole is more than the sum of the parts. We want to avoid a new verbal definition and refer instead to a few excellent papers and books [1, 2, 9, 11].

There seem to be certain necessary ingredients for an observed phenomenon to be called “emergent”: A large population of interacting elements (or agents) without
central control and hence based only on local rules leads to a macroscopic behaviour which displays new properties not existent on the element-level. This macroscopic pattern is perceived as structure or order. Although the resulting order is a necessary precondition for emergence, it is not sufficient. We require that this order has been developed without external intervention—i.e. self-organised. Hence we define emergence as self-organised order.\textsuperscript{1} An attempt to measure emergence quantitatively should therefore rely on a well-known metric for order, i.e. entropy.

Section 2 of this article proposes a method to determine the entropy of an arbitrary system based on Shannon’s entropy definition. This definition relies on the selection of observable attributes, which means a subjective influence on the measured values. Section 3 introduces the notion of an observation model, which subsumes these subjective decisions. Emergence is not the same as entropy. In Sect. 4 we derive an emergence measure based on entropy and discuss its implications in comparison to Shannon’s redundancy. Section 5 discusses the limitations of our approach, the relationship of redundancy and emergence, and the term “pragmatic information” as introduced by von Weizsäcker, Sect. 6 proposes an Observer/Controller architecture we are presently implementing, which includes detectors for the measurement of emergence. Section 7 presents first experimental results of emergence measurements.

2 The Measurement of Order

The meaning of order as perceived by a human\textsuperscript{2} observer is not clear without ambiguity. A homogeneous mixture of two liquids can be regarded as “orderly” (Fig. 1, right). Applying the thermodynamic entropy, however, will result in lower entropy (i.e. higher order) for the example on the left of Fig. 1. Apparently, order depends on the selection of certain attributes by the (human) observer. If we are interested in the spatial structure we have to base our measurement on the positions of the molecules (Fig. 1, left), if we are interested in homogeneity we can use the relative distances between the molecules (Fig. 1, right). The emergence definition presented in this article is based on the statistical definition of entropy (which essentially can be explained as counting events or occurrences).

The computation of the entropy of a system $S$ with $N$ elements $e_i$ is done as follows:

\textsuperscript{1}We appreciate the discussion of the possible separation of emergence and self-organisation by T. De Wolf but claim that for practically interesting phenomena emergence always implies self-organisation.

\textsuperscript{2}Currently the only observers who make these decisions are human designers and researchers, but eventually one could in fact imagine a system that could make these decisions based on knowledge bases and experiments with a target system (e.g. trying out a set of likely candidate attributes, etc.). The theoretical approach of this section is the basis for our observer/controller architectures discussed later in this article.
1. Select an attribute $A$ of the system elements of $S$ with discrete, enumerable values $a_j$.
2. Observe all elements $e_i$ and assign a value $a_j$ to each $e_i$. This step corresponds to a quantisation.
3. Transform into a probability distribution (by considering the relative frequency as a probability) over the attribute values $a_j$ (i.e. a histogram) with $p_j$ being the probability of occurrence of attribute $a_j$ in the ensemble of elements $e_i$.
4. Compute the entropy according to Shannon’s definition

$$H_A = -\sum_{j=0}^{N-1} p_j \cdot \log(p_j)$$

If the attribute values are equally distributed (all $p_j$ equal) we will obtain the maximum entropy. Any deviation from the equal distribution will result in lower entropy values (i.e. higher order). In other words: The more structure is present (unequal distribution), the more order is measured. The unit of measurement is bit/element. So the entropy value can be interpreted as the information content necessary to describe the given system $S$ with regard to attribute $A$. A highly ordered system requires a simpler description than a chaotic one.\(^3\)

### 3 Observation Model

The resulting entropy value depends on two decisions of the observer: (1) Which attribute $A$ is measured? and (2) With what resolution (or quantisation) is $A$ measured? The quantisation determines the information content of the system description but it is not a property of the system. Neither is the selection of a certain attribute $A$ a system property. This means that a measured entropy value is only meaningful if we know the exact observation context. This context is subsumed by the observation model.

This reflects the fact that order is not an intrinsic property of the system. Rather order depends on subjective decisions or capabilities of the observer. In living systems the sensory equipment limits the selection of observable attributes and the

\(^3\)This reminds of the definition of Kolmogorov complexity [5].
resolution of the measurement. In addition, the brain directs attention to certain observables, which are relevant in the present situation, and masks other attributes or registers them with lower effort, i.e. lower resolution. Hence, order results from an interaction between the observer and the observed system guided by the observation model. The observation model depends on the capabilities of the sensory equipment and the utility of certain observations with regard to the purpose.

An observer might be interested in more than one attribute. In this case, we obtain a vector of entropy values \( (H_A, H_B, H_C, \ldots) \) with respect to attributes \( A, B, C, \ldots \). We could add them into a total system entropy \( H_S \). \( H_S \) denotes the information content of the total system description under the given observation models. It has the drawback of hiding or averaging the single attribute entropies. Therefore we prefer the non-added entropy values.

### 4 Emergence

Entropy is not the same as emergence. Entropy decreases with increasing order while emergence should increase with order. As a first try we define emergence as the difference \( \Delta H \) between the entropy at the beginning of some process and at the end:

\[
\Delta H = H_{\text{start}} - H_{\text{end}}
\]

(2)

In case of an increase of order this results in a positive value of \( \Delta H \). A process is called emergent if (1) \( \Delta H > 0 \) and (2) the process is self-organised. This definition has two problems:

1. The measurement of \( H \) depends on the observation model (or abstraction level). An observation on a higher abstraction level will lead to a lower entropy value \( H \) even when there is no change of \( S \) in terms of self-organised order.
2. Since the start condition of the system is arbitrary, \( \Delta H \) represents only a relative value for the increase of order. It would be preferable to have a normalised emergence measure.

The first problem can be solved by determining the portion of \( \Delta H \), which is due to a change of abstraction level \( (\Delta H_{\text{view}}) \) and subtracting it.

\[
\Delta H = \Delta H_{\text{emergence}} + \Delta H_{\text{view}}
\]

(3)

\[
\Delta H_{\text{emergence}} = \Delta H - \Delta H_{\text{view}}
\]

(4)

In other words: Equation (1) holds only if we have not changed the abstraction level or if \( \Delta H_{\text{view}} \) can be determined and subtracted.

The second problem is solved by definition of an absolute reference as starting condition. Obviously this starting condition could be the state of maximum disorder with an entropy of \( H_{\text{max}} \). \( H_{\text{max}} \) corresponds to the equal probability distribution.
This leads to the following definition:

$$M = \Delta H_{\text{emergence}} = H_{\text{max}} - H - \Delta H_{\text{view}}$$  \hspace{1cm} (5)

Absolute emergence is the increase of order due to self-organised processes between the elements of a system $S$ in relation to a starting condition of maximal disorder.

The observation model used for both observations must be the same ($\Delta H_{\text{view}} = 0$) or $\Delta H_{\text{view}}$ must be determined and subtracted.

We can also define the relative emergence $m$ as

$$m = \frac{H_{\text{max}} - H}{H_{\text{max}}} \quad \text{(if } \Delta H_{\text{view}} = 0)$$  \hspace{1cm} (6)

$m$ has a value between 0 and 1, $m = 0$ means high disorder and $m = 1$ high order.

Of course we can define—in analogy to the above discussion—attribute-specific emergence-values $M_A$, $M_B$, $M_C$ or $m_A$, $m_B$, $m_C$. The vector of $M_k$ or $m_k$ (with $k$ denoting the attributes of the elements of a given system) constitutes a so-called emergence fingerprint. An emergence fingerprint can be visualised e.g. as a 6-dimensional Kiviat graph (Fig. 2). It represents the “order pattern” with regard to the chosen attributes. In the example of Fig. 2, the 6 attributes $x$ and $y$ coordinate, status, intention (of an animat), direction of movement and colour have been chosen. The fingerprint changes over time ($t_0, t_1, t_2$) and thus represents the development of order in the different dimensions (attributes).

5 Discussion

5.1 Limitations

The definition is not applicable if $\Delta H_{\text{view}}$ cannot be made zero or determined quantitatively. This is always the case if the macro phenomenon is totally different from the micro behaviours as seemingly in the case of the resonance frequency as an emergent property resulting from the interaction of a capacitor and an inductivity. Our quantitative definition of emergence is based on the assumption that emergent phenomena can always be observed in terms of patterns (space and/or time) consisting of large ensembles of elements. The resonance frequency of a resonant circuit
does not constitute the emergent pattern but is rather a property of such a pattern. Order can also be determined in the time or frequency domain. Therefore, we can apply our emergence definition to the resonance frequency example if we observe the system behaviour after a Fourier analysis. This extends the above definition of the observer model: Any type of preprocessing can also be a part of the observer model. This corresponds quite well to the operation of the animal (and human) perception.

We admit that our model does not cover the so-called ‘strong emergence’ definition, which demands that emergence is a phenomenon principally unexplainable. But this is a quite unscientific argumentation, which we reject. To the contrary, we would like to propose, that only a quantifiable phenomenon resulting in a (self-organised) increase of order, deserves to be accepted as emergence. If this definition is too restrictive, excluding some unexplainable emergent effects, we could accept that what we measure with our method is “quantitative emergence” and constitutes a certain form of emergence meaningful in technical systems. This definition leaves room for wider definitions of emergence in a more general meaning.

5.2 Redundancy and Emergence

The reader familiar with Shannon’s information theory might have recognised that our definition of emergence is formally equivalent to Shannon’s redundancy since redundancy

$$R = H_{\text{max}} - H$$  \hspace{1cm} (7)

and relative redundancy

$$r = \frac{H_{\text{max}} - H}{H_{\text{max}}}.$$  \hspace{1cm} (8)

Redundancy is a property of a message source and a code, which should be reduced as far as possible in order to utilise a given channel optimally. Emergence on the other hand is a measure of order in a system, which in many cases is highly desirable. At least, as we have seen, a high emergence value means that the description complexity of the system is low. The explanation of the apparent contradiction lies in the different points of view of a communication engineer (Shannon) and a systems engineer.

A channel is utilised optimally if it transports only information, which is new to the receiver. Shannon defines predictable information as useless or redundant. But the notion that predictable information is useless contradicts both our intuition and biological research. It is justified only in communications engineering. Shannon assumes that sender and receiver have the same semantic framework, the receiver has a-priori knowledge, he can match a received message against a known

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4In the cochlea, the sound moves hair bundles, which respond to certain frequencies. The brain therefore reacts to preprocessed signals [10].
set of messages. A received message has a value only within this pre-defined context. The common context in a traditional technical system is given by the fact that both, sender and receiver (and the communication system in between), have been designed by the “same” engineer.

This is not true for adaptive systems (living systems or organic computing systems). Communicating adaptive systems have (1) to maintain a valid context, which might change over time, and (2) to validate new information within this context. The maintenance of context requires regular affirmation and a certain degree of modification. The more context the receiver has accumulated in his memory (be it genetic or acquired), the less new or current information he needs in order to recognize a certain situation and act accordingly.

This means that each animal (or animat) will strive to build and maintain a dependable and stable context “database” which allows it to reduce the amount of new information to be transferred in a possibly dangerous situation and hence save valuable time and energy. This works fine as long as the environment is stable and in agreement with the memorized context database. In changing situations this agreement will be disturbed, and erroneous decisions will follow. The context information has to be updated as fast as possible.

The above discussion suggests a possible approach to separate information into two parts: affirmation information and newness information. The first part is used to affirm and/or modify the context database, the second selects between several contexts. But it is more realistic to assume that each received message is used as a whole for both purposes: It is compared to the stored contexts and selects one of them in case there is a sufficient match. A successful match results in an action (stimulus-response relationship). If there is no match or if there is a wrong match, a detrimental action will result leading to the necessity to update the context database. An update phase will require the temporary collection of more information until the animat can return to business as usual.

An animat (animal) prefers a low entropy environment (orderly and predictable, with high emergence)! On the other hand, instability (caused by random processes like mutation) is the necessary precondition for the exploration of unknown areas of the configuration space.

5.3 Pragmatic Information

The term pragmatic information has been introduced by Christine and Ernst von Weizsäcker, cited by Küppers in [4]. Pragmatic information means information, which has an effect on the receiver. This could be a structural change (leading to some kind of action) or the readiness of the receiver for some action ([4, p. 85]).

Animats are artificial animals, e.g. robots equipped with sensors, actuators and some decision mechanism.
Pragmatic information is determined by two aspects: Newness (German: *Erstmaligkeit*) and affirmation (German: *Bestätigung*). A qualitative curve proposed by Christine and Ernst von Weizsäcker (Fig. 3) claims that the Shannon part of the information measures newness. It is zero for a known (i.e. predicted) event and increases with increasing newness. On the other hand, for a message to be recognised within a context there must be a-priori knowledge of this context. Therefore, pragmatic information is also zero if there has been no prior communication between sender and receiver. In this case, newness = 100%. Affirmation is complementary to newness: It is zero when a message is totally new, and 100% when a message is known in advance. If there are two zero values of pragmatic information, there must be at least one maximum in between. Von Weizsäcker concludes that to achieve a maximum of pragmatic information, there must be a certain optimal combination of newness and affirmation in the communication relation between two partners.

A highly predictable system as message source (i.e. a system displaying high order = high emergence) requires a channel with low bandwidth because the transmitted information serves essentially as affirmation, which is needed less frequently. The receiver compares these messages to his context database, finds matches with a high probability and initiates corresponding actions. If affirmation is sent too frequently, it becomes useless: the channel transports redundant information (already known by the receiver). As soon as the message source changes its behaviour, the receiver needs more frequent update information in order to change his context database. The newness aspect of the messages becomes more important.

Technically speaking, there are two mechanisms in the receiver working in parallel on all received messages. On the lower “working” level, messages are compared against known probability distributions and mapped to actions. On a higher semantic level, the newness of all messages and the effectiveness of the corresponding actions are monitored. In case of inadequate results, the structure of the receiver has to be adapted by changing/extending the context database and adding new actions. This higher level is realised by (possibly multi-level) Observer/Controller architectures as shown in the next section. The lower level corresponds to the production system.
6 Observer/Controller Architecture

The objective of our work is to make emergent phenomena accessible to a practical measurement process. This is important in technical applications that have to detect emergent phenomena in order to support or to suppress them. In other projects presently run by the authors the collective behaviour of chicken in a chicken farm [3], the behaviour of cars in the environment of an intersection [7] or the synchronisation of elevators (so-called Bunching effect [3]) is of interest. We propose a generalised Observer/Controller architecture [8] (Fig. 4). The observer collects and aggregates information about the production system. The aggregated values (system indicators) are reported to the controller who takes appropriate actions to influence the production system. The observer contains several specialised detectors to calculate the system indicators from the observed raw data (Fig. 5). We are presently building emergence detectors specialised for certain attributes. The collection of attribute emergence values (the emergence fingerprint) is a part of the observation

Fig. 4 Observer/controller architecture

Fig. 5 Observer architecture
result as determined by the observer. The observer model influences the observation procedure, e.g. by selecting certain detectors or certain attributes of interest. The feedback from the controller to the observer directs attention to certain observables of interest in the current context.

7 Experimental Results

In this section we present first experimental results of the emergence fingerprint. The results discussed here could be part of the observer in the Observer/Controller architecture presented in Sect. 6.

7.1 Experimental Environment

One of the experimental environments is a chicken simulator, whose goal is to explain the collective cannibalistic behaviour of densely packed chicken in cages (cooperation with the University of Veterinary Medicine Hannover). This behaviour is frequently observed when a chicken is injured, and leads to a major loss of animals. The described reaction occurs only on the basis of an optical stimulus. That means the reaction exists as long as the stimulus is apparent. While simulating this behaviour, order patterns emerge in form of chicken swarms. These patterns are at present interpreted by human experts. It should be possible to classify them automatically. The emergent behaviour in this scenario is spatial, but swarms move over time. This is a case of “negative”, i.e. unwanted, emergence, since the global goal is to reduce chicken death rate. The controller has to react with actions to disperse the swarms.

7.2 Results

Figures 6, 7 and 8 show three typical states of emergent clustering behaviours (taken from our simulations). State 1 shows no recognisable clustering. In state 2, a chicken is wounded, and a small group of aggressing chicken has already clustered around it. In state 3, all the chicken of the cage have realised the injury and are participating in the attack. The Kiviat graph next to the state pictures has three dimensions: $x$- and $y$-coordinate and the direction (of view). Only the emergence values for the $x$- and the $y$-coordinate, $m_x$ and $m_y$, show a significant increase (as expected). The heading direction plays no role in the chasing behaviour. The corresponding emergence $m_d$ stays very small. Figure 9 shows the overlay of the 3 states and their development over time.
1.2 Quantitative Emergence

Fig. 6 Emergence fingerprint of state 1: no cluster; $m_x = 0.181$, $m_y = 0.177$, $m_d = 0.091$ (uninjured chicken: white, feeding troughs: hexagons)

Fig. 7 Emergence fingerprint of state 2: small cluster; $m_x = 0.226$, $m_y = 0.237$, $m_d = 0.046$

Fig. 8 Emergence fingerprint of state 3: one big cluster; $m_x = 0.359$, $m_y = 0.328$, $m_d = 0.041$
We are especially interested in the prediction of future emergent behaviour in order to be able to prevent unwanted behaviour in time. To deal with this goal, it must be possible to predict the positions of the chicken. This can be done by extrapolating a trajectory. We measure the position of the chicken at two consecutive points in time. Based on these two points the trajectory of the chicken is computed by extending the line between them. Only those positions are of practical interest that can be reached within a certain prediction time $\Delta t_{\text{prediction}}$. Using the present speed of the animals $v_{\text{average}}$, we determine a critical distance $d = v_{\text{average}} \cdot \Delta t_{\text{prediction}}$. We compute the intersection points of all trajectories within the critical distance $d$ (Fig. 10). An accumulation of these points means that the chicken head on a point in the area, which might indicate the existence of an injured chicken (Fig. 11).
1.2 Quantitative Emergence

**Fig. 12** Prediction: The emergence \( m_{x, prediction} \) indicates a future cluster \( (m_x) \), which emerges about 10 time units later.

Point accumulation in space can also be measured by using the emergence indicator applied to their \( x \)- and \( y \)-coordinates. Figure 12 shows the emergence values of the \( x \)-coordinate of the chicken positions and the ones of the intersection points of the trajectories. The emergence of the intersection points grows before the emergence of the actual chicken positions and can therefore be used as an early warning indicator for chicken clustering. We are currently experimenting to increase the prediction time and to reduce the effect of noise.

8 Conclusion and Outlook

We have proposed a quantitative measure of emergence based on the statistical definition of entropy and discussed it in comparison with Shannon’s information theory. While proponents of a so-called strong definition of emergence might argue that “true” emergent effects must always represent something totally new and unexpected, we claim that with our emergence definition we can measure at least some effects of the generation of order. Emergence definitions going beyond might have to live with the flaw that they principally do not lend themselves to a quantitative approach.

It is the objective of our work to make emergent effects quantitatively treatable in a technical environment. We have proposed an observer/controller architecture with special detectors for determining attribute emergence values. First experimental results obtained from a chicken simulation show the viability of the approach.

We plan to extend these detectors by preprocessing steps, using perhaps a Fourier analysis to make regular patterns like a crystal lattice treatable. The method will be applied to more technical problems like self-synchronising elevators (bunching effect) and traffic simulations.

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