$= 1 - P(\text{all } n \text{ components succeed}) = 1 - (1-p)^n$. More generally, in terms of component reliabilities $(p_1, \ldots, p_n)$, the system reliability function $\Psi$ is

$$\Psi(p_1, \ldots, p_n) = \prod_{i=1}^{n}(1 - p_i)\ . \tag{1.16}$$

A *parallel* system is just the opposite; it fails only after every one of its $n$ working components fail. The system failure probability is then

$$\Psi(p_1, \ldots, p_n) = 1 - \prod_{i=1}^{n} p_i\ . \tag{1.17}$$

The parallel system and series system are special cases of a $k$-out-of-$n$ system, which is a system that works as long as at least $k$ out of its $n$ components work. Assuming $p_i = p$, $i = 1, \ldots, n$, the reliability of a $k$-out-of-$n$ systems is

$$\Psi(p) = \sum_{i=k}^{n} \binom{n}{i} (1-p)^i p^{n-1}\ . \tag{1.18}$$

Of course, most component arrangements are much more complex that a series or parallel system. With just three components, there are five unique ways of arranging the components in a coherent way (that is, so that each component success contributes positively to the system reliability). Figure 1.3 shows the *system structure* of those five arrangements in terms of a *logic diagram* including a series system (1), a 2-out-of-3 system (3), and a parallel system (5). Note that the 2-out-of-3 system cannot be diagrammed with only three components, so each component is represented twice in the logic diagram. Figure 1.4 displays the corresponding reliabilities, as a function of the component reliability $0 \le p \le 1$ of those five systems. Fundamental properties of coherent systems are discussed in [1.2] and [1.4].

### 1.4.1 Estimating System and Component Reliability

In many complex systems, the reliability of the system can be computed through the reliability of the components along with the system's structure function. If the exact reliability is too difficult to compute explicitly, reliability bounds might be achievable based on *minimum cut sets* (MCS) and *minimum path sets* (MPS). An MPS is the collection of the smallest component sets that are required to work in order to keep the system working. An MCS is the collection of the smallest component sets that are required to fail in order for the system to fail. Table 1.2 shows the minimum cuts sets and path sets for the three-component systems from Fig. 1.3.

In most industrial systems, components have different roles and varying reliabilities, and often the component reliability depends on the working status of other components. System reliability can be simplified through fault-tree analyses (see Chapt. 7 of [1.6], for example), but uncertainty bounds for system reliability are typically determined through simulation.

In laboratory tests, component reliabilities are determined and the system reliability is computed as

**Table 1.2** Minimum cut sets and path sets for the systems in Fig. 1.3

| System | Minimum path sets | Minimum cut sets |
|---|---|---|
| 1 | {A,B,C} | {A}, {B}, {C} |
| 2 | {A,B}, {C} | {A,C}, {B,C} |
| 3 | {A,B}, {A,C}, {B,C} | {A,B}, {A,C}, {B,C} |
| 4 | {A,B}, {A,C} | {A}, {B,C} |
| 5 | {A}, {B}, {C} | {A,B,C} |

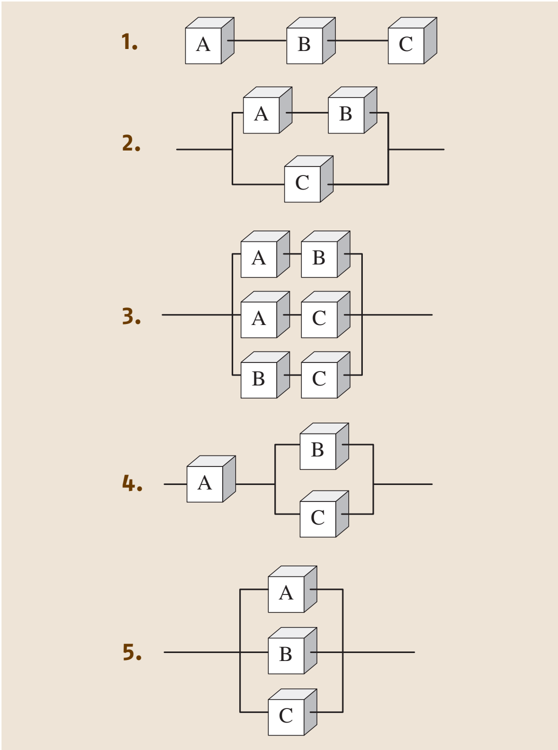


**Fig. 1.3** Five unique systems of three components: (1) is series, (3) is 2-out-of-3 and (5) is parallel

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