

Contents

- 1 Introduction 1**
- 1.1 What this Book Is All About 1
- 1.2 What Is Vision? 2
- 1.3 The Magic of Your Visual System 3
- 1.4 Importance of Prior Information 7
 - 1.4.1 Ecological Adaptation Provides Prior Information 7
 - 1.4.2 Generative Models and Latent Quantities 8
 - 1.4.3 Projection onto the Retina Loses Information 9
 - 1.4.4 Bayesian Inference and Priors 9
- 1.5 Natural Images 10
 - 1.5.1 The Image Space 10
 - 1.5.2 Definition of Natural Images 11
- 1.6 Redundancy and Information 13
 - 1.6.1 Information Theory and Image Coding 13
 - 1.6.2 Redundancy Reduction and Neural Coding 14
- 1.7 Statistical Modeling of the Visual System 15
 - 1.7.1 Connecting Information Theory and Bayesian Inference . . 15
 - 1.7.2 Normative vs. Descriptive Modeling of Visual System . . . 15
 - 1.7.3 Toward Predictive Theoretical Neuroscience 16
- 1.8 Features and Statistical Models of Natural Images 17
 - 1.8.1 Image Representations and Features 17
 - 1.8.2 Statistics of Features 18
 - 1.8.3 From Features to Statistical Models 19
- 1.9 The Statistical–Ecological Approach Recapitulated 20
- 1.10 References 21

Part I Background

- 2 Linear Filters and Frequency Analysis 25**
- 2.1 Linear Filtering 25
 - 2.1.1 Definition 25
 - 2.1.2 Impulse Response and Convolution 28
- 2.2 Frequency-Based Representation 29
 - 2.2.1 Motivation 29
 - 2.2.2 Representation in One and Two Dimensions 29
 - 2.2.3 Frequency-Based Representation and Linear Filtering 34
 - 2.2.4 Computation and Mathematical Details 37
- 2.3 Representation Using Linear Basis 38
 - 2.3.1 Basic Idea 38
 - 2.3.2 Frequency-Based Representation as a Basis 40

- 2.4 Space-Frequency Analysis 41
 - 2.4.1 Introduction 41
 - 2.4.2 Space-Frequency Analysis and Gabor Filters 43
 - 2.4.3 Spatial Localization vs. Spectral Accuracy 46
- 2.5 References 48
- 2.6 Exercises 48

- 3 Outline of the Visual System 51**
 - 3.1 Neurons and Firing Rates 51
 - 3.2 From the Eye to the Cortex 53
 - 3.3 Linear Models of Visual Neurons 54
 - 3.3.1 Responses to Visual Stimulation 54
 - 3.3.2 Simple Cells and Linear Models 56
 - 3.3.3 Gabor Models and Selectivities of Simple Cells 57
 - 3.3.4 Frequency Channels 58
 - 3.4 Non-linear Models of Visual Neurons 59
 - 3.4.1 Non-linearities in Simple-Cell Responses 59
 - 3.4.2 Complex Cells and Energy Models 61
 - 3.5 Interactions between Visual Neurons 62
 - 3.6 Topographic Organization 64
 - 3.7 Processing after the Primary Visual Cortex 64
 - 3.8 References 65
 - 3.9 Exercises 65

- 4 Multivariate Probability and Statistics 67**
 - 4.1 Natural Images Patches as Random Vectors 67
 - 4.2 Multivariate Probability Distributions 68
 - 4.2.1 Notation and Motivation 68
 - 4.2.2 Probability Density Function 69
 - 4.3 Marginal and Joint Probabilities 70
 - 4.4 Conditional Probabilities 73
 - 4.5 Independence 75
 - 4.6 Expectation and Covariance 77
 - 4.6.1 Expectation 77
 - 4.6.2 Variance and Covariance in One Dimension 78
 - 4.6.3 Covariance Matrix 78
 - 4.6.4 Independence and Covariances 79
 - 4.7 Bayesian Inference 81
 - 4.7.1 Motivating Example 81
 - 4.7.2 Bayes' Rule 83
 - 4.7.3 Non-informative Priors 83
 - 4.7.4 Bayesian Inference as an Incremental Learning Process 84
 - 4.8 Parameter Estimation and Likelihood 86
 - 4.8.1 Models, Estimation, and Samples 86
 - 4.8.2 Maximum Likelihood and Maximum a Posteriori 87
 - 4.8.3 Prior and Large Samples 89

4.9 References 89

4.10 Exercises 89

Part II Statistics of Linear Features

5 Principal Components and Whitening 93

5.1 DC Component or Mean Grey-Scale Value 93

5.2 Principal Component Analysis 94

5.2.1 A Basic Dependency of Pixels in Natural Images 94

5.2.2 Learning One Feature by Maximization of Variance 96

5.2.3 Learning Many Features by PCA 98

5.2.4 Computational Implementation of PCA 101

5.2.5 The Implications of Translation-Invariance 102

5.3 PCA as a Preprocessing Tool 103

5.3.1 Dimension Reduction by PCA 103

5.3.2 Whitening by PCA 104

5.3.3 Anti-aliasing by PCA 106

5.4 Canonical Preprocessing Used in This Book 109

5.5 Gaussianity as the Basis for PCA 109

5.5.1 The Probability Model Related to PCA 109

5.5.2 PCA as a Generative Model 110

5.5.3 Image Synthesis Results 111

5.6 Power Spectrum of Natural Images 111

5.6.1 The $1/f$ Fourier Amplitude or $1/f^2$ Power Spectrum 111

5.6.2 Connection between Power Spectrum and Covariances 113

5.6.3 Relative Importance of Amplitude and Phase 114

5.7 Anisotropy in Natural Images 115

5.8 Mathematics of Principal Component Analysis* 116

5.8.1 Eigenvalue Decomposition of the Covariance Matrix 117

5.8.2 Eigenvectors and Translation-Invariance 119

5.9 Decorrelation Models of Retina and LGN * 120

5.9.1 Whitening and Redundancy Reduction 120

5.9.2 Patch-Based Decorrelation 121

5.9.3 Filter-Based Decorrelation 124

5.10 Concluding Remarks and References 128

5.11 Exercises 129

6 Sparse Coding and Simple Cells 131

6.1 Definition of Sparseness 131

6.2 Learning One Feature by Maximization of Sparseness 132

6.2.1 Measuring Sparseness: General Framework 133

6.2.2 Measuring Sparseness Using Kurtosis 133

6.2.3 Measuring Sparseness Using Convex Functions of Square 134

6.2.4 The Case of Canonically Preprocessed Data 138

6.2.5 One Feature Learned from Natural Images 138

- 6.3 Learning Many Features by Maximization of Sparseness 139
 - 6.3.1 Deflationary Decorrelation 140
 - 6.3.2 Symmetric Decorrelation 141
 - 6.3.3 Sparseness of Feature vs. Sparseness of Representation . . 141
- 6.4 Sparse Coding Features for Natural Images 143
 - 6.4.1 Full Set of Features 143
 - 6.4.2 Analysis of Tuning Properties 144
- 6.5 How Is Sparseness Useful? 147
 - 6.5.1 Bayesian Modeling 147
 - 6.5.2 Neural Modeling 148
 - 6.5.3 Metabolic Economy 148
- 6.6 Concluding Remarks and References 148
- 6.7 Exercises 149

- 7 Independent Component Analysis 151**
 - 7.1 Limitations of the Sparse Coding Approach 151
 - 7.2 Definition of ICA 152
 - 7.2.1 Independence 152
 - 7.2.2 Generative Model 152
 - 7.2.3 Model for Preprocessed Data 154
 - 7.3 Insufficiency of Second-Order Information 154
 - 7.3.1 Why Whitening Does Not Find Independent Components . 154
 - 7.3.2 Why Components Have to Be Non-Gaussian 156
 - 7.4 The Probability Density Defined by ICA 158
 - 7.5 Maximum Likelihood Estimation in ICA 159
 - 7.6 Results on Natural Images 160
 - 7.6.1 Estimation of Features 160
 - 7.6.2 Image Synthesis Using ICA 160
 - 7.7 Connection to Maximization of Sparseness 161
 - 7.7.1 Likelihood as a Measure of Sparseness 161
 - 7.7.2 Optimal Sparseness Measures 163
 - 7.8 Why Are Independent Components Sparse? 166
 - 7.8.1 Different Forms of Non-Gaussianity 167
 - 7.8.2 Non-Gaussianity in Natural Images 167
 - 7.8.3 Why Is Sparseness Dominant? 168
 - 7.9 General ICA as Maximization of Non-Gaussianity 168
 - 7.9.1 Central Limit Theorem 169
 - 7.9.2 “Non-Gaussian Is Independent” 169
 - 7.9.3 Sparse Coding as a Special Case of ICA 170
 - 7.10 Receptive Fields vs. Feature Vectors 171
 - 7.11 Problem of Inversion of Preprocessing 172
 - 7.12 Frequency Channels and ICA 173
 - 7.13 Concluding Remarks and References 173
 - 7.14 Exercises 174

8 Information-Theoretic Interpretations 177

8.1 Basic Motivation for Information Theory 177

 8.1.1 Compression 177

 8.1.2 Transmission 178

8.2 Entropy as a Measure of Uncertainty 179

 8.2.1 Definition of Entropy 179

 8.2.2 Entropy as Minimum Coding Length 180

 8.2.3 Redundancy 181

 8.2.4 Differential Entropy 182

 8.2.5 Maximum Entropy 183

8.3 Mutual Information 184

8.4 Minimum Entropy Coding of Natural Images 185

 8.4.1 Image Compression and Sparse Coding 185

 8.4.2 Mutual Information and Sparse Coding 187

 8.4.3 Minimum Entropy Coding in the Cortex 187

8.5 Information Transmission in the Nervous System 188

 8.5.1 Definition of Information Flow and Infomax 188

 8.5.2 Basic Infomax with Linear Neurons 188

 8.5.3 Infomax with Non-linear Neurons 189

 8.5.4 Infomax with Non-constant Noise Variance 190

8.6 Caveats in Application of Information Theory 193

8.7 Concluding Remarks and References 195

8.8 Exercises 195

Part III Nonlinear Features and Dependency of Linear Features

9 Energy Correlation of Linear Features and Normalization 199

9.1 Why Estimated Independent Components Are Not Independent 199

 9.1.1 Estimates vs. Theoretical Components 199

 9.1.2 Counting the Number of Free Parameters 200

9.2 Correlations of Squares of Components in Natural Images 201

9.3 Modeling Using a Variance Variable 201

9.4 Normalization of Variance and Contrast Gain Control 203

9.5 Physical and Neurophysiological Interpretations 205

 9.5.1 Canceling the Effect of Changing Lighting Conditions 205

 9.5.2 Uniform Surfaces 206

 9.5.3 Saturation of Cell Responses 206

9.6 Effect of Normalization on ICA 207

9.7 Concluding Remarks and References 210

9.8 Exercises 211

10 Energy Detectors and Complex Cells 213

10.1 Subspace Model of Invariant Features 213

 10.1.1 Why Linear Features Are Insufficient 213

 10.1.2 Subspaces or Groups of Linear Features 213

 10.1.3 Energy Model of Feature Detection 214

10.2	Maximizing Sparseness in the Energy Model	216
10.2.1	Definition of Sparseness of Output	216
10.2.2	One Feature Learned from Natural Images	217
10.3	Model of Independent Subspace Analysis	219
10.4	Dependency as Energy Correlation	220
10.4.1	Why Energy Correlations Are Related to Sparseness	220
10.4.2	Spherical Symmetry and Changing Variance	221
10.4.3	Correlation of Squares and Convexity of Non-linearity	222
10.5	Connection to Contrast Gain Control	223
10.6	ISA as a Non-linear Version of ICA	224
10.7	Results on Natural Images	225
10.7.1	Emergence of Invariance to Phase	225
10.7.2	The Importance of Being Invariant	230
10.7.3	Grouping of Dependencies	232
10.7.4	Superiority of the Model over ICA	232
10.8	Analysis of Convexity and Energy Correlations*	234
10.8.1	Variance Variable Model Gives Convex h	234
10.8.2	Convex h Typically Implies Positive Energy Correlations	235
10.9	Concluding Remarks and References	236
10.10	Exercises	236
11	Energy Correlations and Topographic Organization	239
11.1	Topography in the Cortex	239
11.2	Modeling Topography by Statistical Dependence	240
11.2.1	Topographic Grid	240
11.2.2	Defining Topography by Statistical Dependencies	240
11.3	Definition of Topographic ICA	242
11.4	Connection to Independent Subspaces and Invariant Features	243
11.5	Utility of Topography	244
11.6	Estimation of Topographic ICA	245
11.7	Topographic ICA of Natural Images	246
11.7.1	Emergence of V1-like Topography	246
11.7.2	Comparison with Other Models	253
11.8	Learning Both Layers in a Two-Layer Model *	253
11.8.1	Generative vs. Energy-Based Approach	253
11.8.2	Definition of the Generative Model	254
11.8.3	Basic Properties of the Generative Model	255
11.8.4	Estimation of the Generative Model	256
11.8.5	Energy-Based Two-Layer Models	259
11.9	Concluding Remarks and References	260
12	Dependencies of Energy Detectors: Beyond V1	263
12.1	Predictive Modeling of Extrastriate Cortex	263
12.2	Simulation of V1 by a Fixed Two-Layer Model	263
12.3	Learning the Third Layer by Another ICA Model	265

- 12.4 Methods for Analyzing Higher-Order Components 266
- 12.5 Results on Natural Images 268
 - 12.5.1 Emergence of Collinear Contour Units 268
 - 12.5.2 Emergence of Pooling over Frequencies 269
- 12.6 Discussion of Results 273
 - 12.6.1 Why Coding of Contours? 273
 - 12.6.2 Frequency Channels and Edges 274
 - 12.6.3 Toward Predictive Modeling 274
 - 12.6.4 References and Related Work 275
- 12.7 Conclusion 276
- 13 Overcomplete and Non-negative Models 277**
 - 13.1 Overcomplete Bases 277
 - 13.1.1 Motivation 277
 - 13.1.2 Definition of Generative Model 278
 - 13.1.3 Nonlinear Computation of the Basis Coefficients 279
 - 13.1.4 Estimation of the Basis 281
 - 13.1.5 Approach Using Energy-Based Models 282
 - 13.1.6 Results on Natural Images 285
 - 13.1.7 Markov Random Field Models * 285
 - 13.2 Non-negative Models 288
 - 13.2.1 Motivation 288
 - 13.2.2 Definition 288
 - 13.2.3 Adding Sparseness Constraints 290
 - 13.3 Conclusion 293
- 14 Lateral Interactions and Feedback 295**
 - 14.1 Feedback as Bayesian Inference 295
 - 14.1.1 Example: Contour Integrator Units 296
 - 14.1.2 Thresholding (Shrinkage) of a Sparse Code 298
 - 14.1.3 Categorization and Top-Down Feedback 302
 - 14.2 Overcomplete Basis and End-stopping 302
 - 14.3 Predictive Coding 304
 - 14.4 Conclusion 305

Part IV Time, Color, and Stereo

- 15 Color and Stereo Images 309**
 - 15.1 Color Image Experiments 309
 - 15.1.1 Choice of Data 309
 - 15.1.2 Preprocessing and PCA 310
 - 15.1.3 ICA Results and Discussion 313
 - 15.2 Stereo Image Experiments 315
 - 15.2.1 Choice of Data 315
 - 15.2.2 Preprocessing and PCA 316
 - 15.2.3 ICA Results and Discussion 317

- 15.3 Further References 322
 - 15.3.1 Color and Stereo Images 322
 - 15.3.2 Other Modalities, Including Audition 323
- 15.4 Conclusion 323
- 16 Temporal Sequences of Natural Images 325**
 - 16.1 Natural Image Sequences and Spatiotemporal Filtering 325
 - 16.2 Temporal and Spatiotemporal Receptive Fields 326
 - 16.3 Second-Order Statistics 328
 - 16.3.1 Average Spatiotemporal Power Spectrum 328
 - 16.3.2 The Temporally Decorrelating Filter 332
 - 16.4 Sparse Coding and ICA of Natural Image Sequences 333
 - 16.5 Temporal Coherence in Spatial Features 336
 - 16.5.1 Temporal Coherence and Invariant Representation 336
 - 16.5.2 Quantifying Temporal Coherence 337
 - 16.5.3 Interpretation as Generative Model * 338
 - 16.5.4 Experiments on Natural Image Sequences 339
 - 16.5.5 Why Gabor-Like Features Maximize Temporal Coherence . 341
 - 16.5.6 Control Experiments 344
 - 16.6 Spatiotemporal Energy Correlations in Linear Features 345
 - 16.6.1 Definition of the Model 345
 - 16.6.2 Estimation of the Model 347
 - 16.6.3 Experiments on Natural Images 348
 - 16.6.4 Intuitive Explanation of Results 350
 - 16.7 Unifying Model of Spatiotemporal Dependencies 352
 - 16.8 Features with Minimal Average Temporal Change 354
 - 16.8.1 Slow Feature Analysis 354
 - 16.8.2 Quadratic Slow Feature Analysis 357
 - 16.8.3 Sparse Slow Feature Analysis 359
 - 16.9 Conclusion 361

Part V Conclusion

- 17 Conclusion and Future Prospects 365**
 - 17.1 Short Overview 365
 - 17.2 Open, or Frequently Asked, Questions 367
 - 17.2.1 What Is the Real Learning Principle in the Brain? 367
 - 17.2.2 Nature vs. Nurture 368
 - 17.2.3 How to Model Whole Images 369
 - 17.2.4 Are There Clear-Cut Cell Types? 369
 - 17.2.5 How Far Can We Go? 371
 - 17.3 Other Mathematical Models of Images 371
 - 17.3.1 Scaling Laws 372
 - 17.3.2 Wavelet Theory 372
 - 17.3.3 Physically Inspired Models 373
 - 17.4 Future Work 374

Part VI Appendix: Supplementary Mathematical Tools

18 Optimization Theory and Algorithms 377

18.1 Levels of Modeling 377

18.2 Gradient Method 378

 18.2.1 Definition and Meaning of Gradient 378

 18.2.2 Gradient and Optimization 380

 18.2.3 Optimization of Function of Matrix 381

 18.2.4 Constrained Optimization 381

18.3 Global and Local Maxima 383

18.4 Hebb’s Rule and Gradient Methods 384

 18.4.1 Hebb’s Rule 384

 18.4.2 Hebb’s Rule and Optimization 385

 18.4.3 Stochastic Gradient Methods 386

 18.4.4 Role of the Hebbian Non-linearity 387

 18.4.5 Receptive Fields vs. Synaptic Strengths 388

 18.4.6 The Problem of Feedback 388

18.5 Optimization in Topographic ICA * 389

18.6 Beyond Basic Gradient Methods * 390

 18.6.1 Newton’s Method 391

 18.6.2 Conjugate Gradient Methods 393

18.7 FastICA, a Fixed-Point Algorithm for ICA 394

 18.7.1 The FastICA Algorithm 394

 18.7.2 Choice of the FastICA Non-linearity 395

 18.7.3 Mathematics of FastICA * 395

19 Crash Course on Linear Algebra 399

19.1 Vectors 399

19.2 Linear Transformations 400

19.3 Matrices 401

19.4 Determinant 402

19.5 Inverse 402

19.6 Basis Representations 403

19.7 Orthogonality 404

19.8 Pseudo-Inverse * 405

20 The Discrete Fourier Transform 407

20.1 Linear Shift-Invariant Systems 407

20.2 One-Dimensional Discrete Fourier Transform 408

 20.2.1 Euler’s Formula 408

 20.2.2 Representation in Complex Exponentials 408

 20.2.3 The Discrete Fourier Transform and Its Inverse 411

20.3 Two- and Three-Dimensional Discrete Fourier Transforms 417

21 Estimation of Non-normalized Statistical Models 419

21.1 Non-normalized Statistical Models 419

21.2 Estimation by Score Matching 420

21.3 Example 1: Multivariate Gaussian Density 422

21.4 Example 2: Estimation of Basic ICA Model 424

21.5 Example 3: Estimation of an Overcomplete ICA Model 425

21.6 Conclusion 425

References 427

Index 441



<http://www.springer.com/978-1-84882-490-4>

Natural Image Statistics

A Probabilistic Approach to Early Computational Vision.

Hyvärinen, A.; Hurri, J.; Hoyer, P.O.

2009, XIX, 448 p., Hardcover

ISBN: 978-1-84882-490-4