Chapter 2
Spacecraft Multi-Mission Planning

Abstract For on-orbit operations missions, the multi-mission mode of one or a group of servicers operating on multiple customers will gradually come true. Spacecraft multi-mission planning is founded on one-to-one single mission planning, including far-range orbital maneuver planning and proximity relative motion planning. In this chapter, a multilevel nonlinear planning model, which can be simplified as mission assignment, sequence arrangement, and time distribution, is established and solution strategy explored. For N-to-N spacecraft mission assignment, we assume one servicer only serves one customer, and the integer programming algorithm is adopted. For one-to-N sequence arrangement and time distribution, the HGABB combined by GA and the branch and bound algorithm is applied to a multiple GEO satellites flyaround inspection scenario.

2.1 Problem Formulation

Assume the customers gather in a constellation or in a certain orbit, and the servicers are deployed as a constellation. Since the customers differ in criticality and service imminence, their mission priority also varies. One servicer can serve one or several customers, constrained by mission duration, fuel consumption, and observation and control.

Spacecraft multi-mission planning helps select appropriate servicer for customer so that the missions with higher priority are first addressed; meanwhile time and fuel costs are optimized. First, mission assignment is optimized to generate an objective set for each servicer. Second, mission sequence is such planned that every servicer is appropriately assigned. Third, time distribution is optimized so that the total fuel consumption to serve all customers is minimized under a given sequence.

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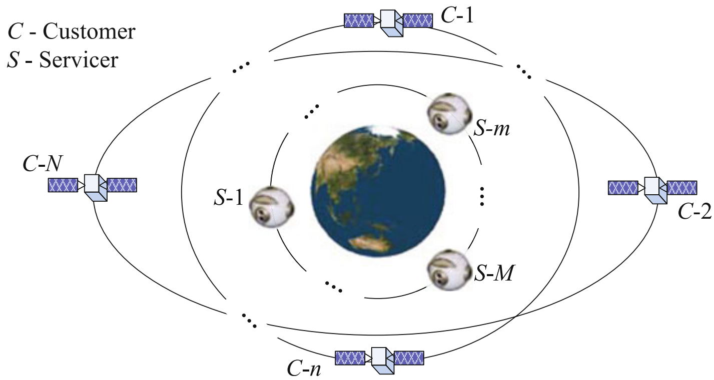

Fig. 2.1 Spacecraft multi-mission planning scenario

To sum up, spacecraft multi-mission planning can be treated as a multi-objective multilevel nonlinear planning problem.

Since time and fuel consumption are much greater in the far-range orbital maneuver phase as compared to the proximity operation phase, the calculation of cost function is based on the far-range orbital maneuver phase here.

2.1.1 Planning Model

In general, the multi-mission planning scenario is defined as in Fig. 2.1, where there are N customers and M servicers ($M \leq N$). A mathematical model is then established as below [1].

2.1.1.1 Decision Variables

For spacecraft multi-mission planning, three kinds of decision variables need to be defined.

1. Mission assignment decision variables
   The mission assignment decision variables are defined as

$$\mathbf{X} = \begin{bmatrix} x_{11} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots \\ x_{M1} & \cdots & x_{MN} \end{bmatrix} \tag{2.1}$$

where

$$x_{mn} = \begin{cases} 1 & \text{assign servicer } m \text{ to serve customer } n \\ 0 & \text{otherwise} \end{cases} \quad m = 1, \ldots, M; \;\; n = 1, \ldots, N \tag{2.2}$$

by which mission assignment is transformed to a 0–1 integer programming problem.

2. Mission sequence decision variables
   The mission sequence decision variables are defined as

$$\mathbf{Q} = [\,Q_1 \quad \cdots \quad Q_M\,]^{\mathrm{T}},\; Q_m = \{s_{m1}, s_{m2}, \ldots, s_{ma_m}\} \tag{2.3}$$

   where Q_m refers to the mission sequence set of servicer m, a_m the number of missions assigned to servicer m, and $s_{mk}(k = 1, \ldots, a_m)$ the index of customer when servicer m performs the kth mission.
3. Time distribution decision variables
   The time distribution decision variables are defined as

$$\mathbf{T} = [\,T_1 \quad \cdots \quad T_M\,]^{\mathrm{T}},\; T_m = [\,\Delta t_{m1} \quad \Delta t_{m2} \quad \cdots \quad \Delta t_{ma_m}\,] \tag{2.4}$$

   where T_m denotes the time distribution scheme of Q_m, and Δt_{mk} the duration of the kth mission.

2.1.1.2 Cost Function

According to the mission requirements, four optimization criteria, maximum priority, minimum fuel, minimum time, and fuel consumption equilibrium among servicers, are selected.

Defining p_n as the mission priority of customer n, the cost function of maximum priority can be written as

$$\max J_1 = \sum_{m=1}^{M} \sum_{n=1}^{N} x_{mn} p_n \tag{2.5}$$

Defining ΔV_{mk} as the velocity increment for servicer m to finish the kth mission, ΔV_m the total velocity increment for servicer m to finish the mission sequence Q_m, the cost function of minimum fuel can be written as

$$\min J_2 = \Delta V = \sum_{m=1}^{M} (\Delta V_m) = \sum_{m=1}^{M} \left(\sum_{k=1}^{a_m} \Delta V_{mk} \right) \tag{2.6}$$

where ΔV_{mk} could be solved by the two-impulse multiple-revolution Lambert transfer algorithm (see Sect. 3.1.1).

Defining Δt_m as the total time for servicer m to finish the mission sequence Q_m, then $\Delta t_m = \sum_{k=1}^{a_m} \Delta t_{mk}$, the cost function of minimum time can be written as

$$\min J_3 = \max\{\Delta t_m\} \tag{2.7}$$

Defining C_m as the remaining orbital maneuver capability, indicated by the velocity increment, and $\overline{C}$ as the mean value of all servicers, then $\overline{C} = \sum_{m=1}^{M} C_m / M$, the cost function of fuel consumption equilibrium is written as

$$\min J_4 = \sqrt{\sum_{m=1}^{M} \left(C_m - \overline{C}\right)^2} \tag{2.8}$$

Note that the above criteria are contradictory to some extent. Minimum fuel and minimum time may not be simultaneously satisfied. Therefore, spacecraft multi-mission planning is a multi-objective optimization problem (MOOP).

2.1.1.3 Constraints

1. Mission constraint

   Assume that each mission is accomplished by one servicer, then

$$\begin{cases} \sum_{n=1}^{N} x_{mn} \le a_m & m = 1, \ldots, M \\ \sum_{m=1}^{M} x_{mn} \le 1 & n = 1, \ldots, N \end{cases} \tag{2.9}$$

   Considering the constraints of observation and control, defined by [t^{ocl}, t^{ocr}], the maneuvering moment t_{mk}^{maneuver} of servicer m must be within the time window [t_{mk}^{ocl}, t_{mk}^{ocr}], written as

$$\exists \left[t_{mk}^{\text{ocl}}, t_{mk}^{\text{ocr}}\right], t_{mk}^{\text{maneuver}} \in \left[t_{mk}^{\text{ocl}}, t_{mk}^{\text{ocr}}\right] \tag{2.10}$$

   Defining $\Delta t^{\max}$ as the acceptable maximum far-range maneuver duration, the constraint of mission duration is expressed as

$$\max\{\Delta t_m\} \le \Delta t^{\max} \tag{2.11}$$

2. Resource constraint
   Define $\Delta V_m^{\max}$ as the maximum maneuver capability offered by servicer m, then

$$\Delta V_m \leq \Delta V_m^{\max} \tag{2.12}$$

Considering the duration requirement Δt_{ins} of command injection, the observation and control window $[t_{m0}^{\text{ocl}}, t_{m0}^{\text{ocr}}]$ before t_{m1}^{maneuver} should satisfy

$$\exists \left[t_{m0}^{\text{ocl}}, t_{m0}^{\text{ocr}}\right], t_{m0}^{\text{ocr}} - t_{m0}^{\text{ocl}} \geq \Delta t_{\text{ins}} \tag{2.13}$$

To sum up, a multilevel multi-objective nonlinear planning model is established as

$$(P1)\begin{cases} \max J_1(\mathbf{X}) = \sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn}p_n \\ \min J_2(\mathbf{X},\mathbf{Q},\mathbf{T}) = \sum_{m=1}^{M}\left(\sum_{k=1}^{a_m}\Delta V_{mk}\right) \\ \min J_3(\mathbf{X},\mathbf{Q},\mathbf{T}) = \max\left\{\sum_{k=1}^{a_m}\Delta t_{mk}\right\} \\ \min J_4(\mathbf{X},\mathbf{Q},\mathbf{T}) = \sqrt{\sum_{m=1}^{M}\left(C_m - \overline{C}\right)^2} \end{cases}$$
$$\text{s.t.} \sum_{n=1}^{N} x_{mn} \leq a_m \quad m = 1, \ldots, M$$
$$\sum_{m=1}^{M} x_{mn} \leq 1 \quad n = 1, \ldots, N \tag{2.14}$$

where $\mathbf{Q}$ and $\mathbf{T}$ are the solutions to Eq. (2.15) for each given $\mathbf{X}$.

$$(P2)\begin{cases} \min J_2(\mathbf{Q},\mathbf{T}) = \sum_{m=1}^{M}(\Delta V_m) \\ \min J_3(\mathbf{Q},\mathbf{T}) = \max\{\Delta t_m\} \\ \min J_4(\mathbf{Q},\mathbf{T}) = \sqrt{\sum_{m=1}^{M}\left(C_m - \overline{C}\right)^2} \end{cases} \tag{2.15}$$

where $\mathbf{T}$ is the solution to Eq. (2.16) for each given $\mathbf{X}$ and $\mathbf{Q}$.

$$(P3)\begin{cases} \min\ J_2(\mathbf{T}) = \sum_{m=1}^{M}(\Delta V_m) \\ \min J_3(\mathbf{T}) = \max\{\Delta t_m\} \\ \min J_4(\mathbf{T}) = \sqrt{\sum_{m=1}^{M}\left(C_m - \overline{C}\right)^2} \end{cases}$$
$$\text{s.t. } \Delta V_m \le \Delta V_m^{\max}$$
$$\max\{\Delta t_m\} \le \Delta t^{\max}$$
$$\exists \left[t_{mk}^{\text{ocl}}, t_{mk}^{\text{ocr}}\right],\ t_{mk}^{\text{maneuver}} \in \left[t_{mk}^{\text{ocl}}, t_{mk}^{\text{ocr}}\right]$$
$$\exists \left[t_{m0}^{\text{ocl}}, t_{m0}^{\text{ocr}}\right],\ t_{m0}^{\text{ocr}} - t_{m0}^{\text{ocl}} \ge \Delta t_{\text{ins}} \tag{2.16}$$

Note that Eqs. (2.14)–(2.16) are all MOOPs with conflicting interests, where the achievement of one objective may be at the cost of another and simultaneous optimality can hardly be secured. Trade-offs must be made among objectives to get Pareto-optimal solutions by multi-objective optimization algorithms.

Traditional multi-objective optimization approaches, such as weighted and constraint methods, transform an MOOP to several different single objective optimization problems (SOOP). However, these traditional approaches require separately solving SOOP many times, resulting in heavy computational burden.

Therefore, to ease the computation work by exploiting the correlation of different SOOPs, the Multi-Objective Evolutionary Algorithm (MOEA) is introduced and mostly applied to get Pareto-optimal solutions to an MOOP. Various MOEAs have flourished in literature such as Vector Evaluated Genetic Algorithm (VEGA) [2] and Niched Pareto Genetic Algorithm (NPGA) [3].

Considering the multilevel nature of Eqs. (2.14)–(2.16), it is difficult to generate the solution by MOEA directly. Therefore, we try to simplify the above model of Eqs. (2.14)–(2.16) by analyzing the spacecraft multi-mission properties, and then look for a proper solution strategy.

2.1.2 Solution Strategy

As mentioned above, the spacecraft multi-mission planning could be divided into mission assignment, sequence arrangement, and time distribution. Therefore, an embedded two-level solution scheme is designed. As illustrated in Fig. 2.2, the lower level is to deal with a two-level planning subsystem composed of sequence arrangement and time distribution, while the upper level is to deal with a two-level planning subsystem composed of mission assignment and the lower subsystem.

The above scheme involves multidimensional discrete/continuous variables, generating a large solution space. According to the orbital deployment and maneuver

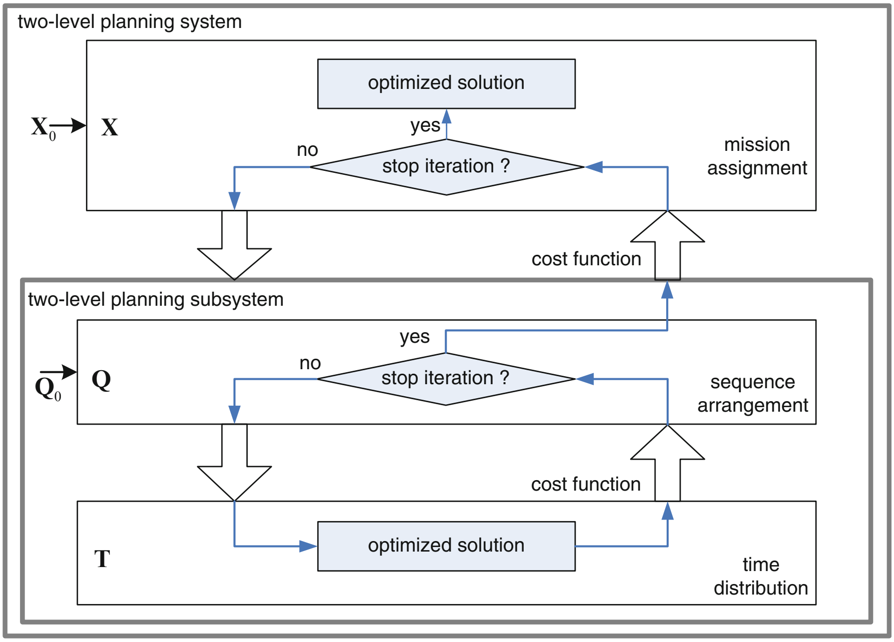

Fig. 2.2 Schematic of spacecraft multi-mission planning model

characteristics, we make proper problem-specific assumptions to explore efficient algorithms.

1. It is difficult for the servicer to make out-of-plane maneuvers due to its limited orbital maneuver capability. If the customers are scattered in different planes, or if the missions are time-demanding, one servicer can only execute one mission. In this way, the model is transformed to a mission assignment problem.
2. If the customers share the same orbit, or if the missions are not time-demanding, one servicer can be given several missions, thus transforming the model to a sequence arrangement and time distribution problem.

2.2 Integer Programming Method for Mission Assignment

Assuming that one servicer only serves one customer, the mission assignment problem is essentially to select a customer for each servicer, thus making sure those with higher priority are first addressed with less time and fuel, and better observation and control.

2.2.1 Planning Model

In Eqs. (2.14)–(2.16), simplifications are made by neglecting the lower-level sequence arrangement and time distribution, represented by $a_m = 1$, to derive the model of mission assignment as

$$
\begin{aligned}
&\begin{cases}
\max J_1(\mathbf{X}) = \sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn} p_n \\
\min J_2(\mathbf{X}) = \sum_{m=1}^{M} \Delta V_{m1} \\
\min J_3(\mathbf{X}) = \max\{\Delta t_{m1}\} \\
\min J_4(\mathbf{X}) = \sqrt{\sum_{m=1}^{M} \left(C_m - \overline{C}\right)^2}
\end{cases} \\
&\text{s.t.} \\
&\quad \sum_{n=1}^{N} x_{mn} \le a_m \quad m = 1, \ldots, M \\
&\quad \sum_{m=1}^{M} x_{mn} \le 1 \quad n = 1, \ldots, N \\
&\quad \Delta V_m \le \Delta V_m^{\max} \\
&\quad \max\{\Delta t_m\} \le \Delta t^{\max} \\
&\quad \exists \left[t_{m1}^{\text{ocl}}, t_{m1}^{\text{ocr}}\right],\ t_{m1}^{\text{maneuver}} \in \left[t_{m1}^{\text{ocl}}, t_{m1}^{\text{ocr}}\right] \\
&\quad \exists \left[t_{m0}^{\text{ocl}}, t_{m0}^{\text{ocr}}\right],\ t_{m0}^{\text{ocr}} - t_{m0}^{\text{ocl}} \ge \Delta t_{\text{ins}}
\end{aligned}
\tag{2.17}
$$

which can be treated as an integer programming problem by separately dealing with maximum and minimum criteria as

$$
\begin{aligned}
&\max J = J_1 \\
&\min J' = \alpha_2 J_2 + \alpha_3 J_3 + \alpha_4 J_4
\end{aligned}
\tag{2.18}
$$

where the first cost function is maximum mission priority, and the second is minimum weighting combination of fuel, time, and fuel consumption equilibrium.

2.2.2 Algorithms

The above model can be solved by the available powerful Mixed Integer Linear Programming (MILP) solvers, such as CPLEX, EXPRESS, and many others. If the mission involves a small number of spacecraft, even the Enumeration

Table 2.1 Initial conditions in simulation

Spacecraft	a (km)	e	i (deg)	Ω (deg)	ω (deg)	f (deg)
C-1	7,250	0.03	94	218	10	15
C-2	7,300	0.01	95	215	20	50
C-3	7,250	0.03	104	220	25	230
C-4	7,300	0.02	103	222	0	130
S-1	7,150	0.01	98	220	30	50
S-2	7,150	0.01	98	220	30	230

Table 2.2 Simulation results

Optimized scheme	J	J_2	J_4
S-1 ~ C-3, S-2 ~ C-4	1.6	2183.4 m/s	1256.5 m/s

method is feasible. Here, we try to integrate stratification method and linear weighting method to transform Eq. (2.18) to a general integer programming model.

First, we find the optimized solutions to the first cost function in Eq. (2.18), thus constituting a set of all feasible solutions, referred to as S_0. Second, we optimize the second cost function in S_0. The detailed steps are given as follows.

Step 1: Order the customers by mission priority, and check whether the first M customers satisfy the constraints. If not, delete them in the customer set and fill the vacancy with subsequent candidates until M qualified customers are found or all customers are checked over. So far we have the optimized solution set to the first cost function.

Step 2: Take the above solution set as the customers and use the branch and bound algorithm to solve the second cost function in Eq. (2.18).

2.2.3 Numerical Simulation

Considering the actual circumstances in OOS, we take $M = 2, N = 4$, and $\Delta t^{\max} = 5$ h. The initial orbital elements are given as in Table 2.1. The tracking, telemetering, and command (TT&C) ground stations include Xi'an, Xiamen, Neimeng, Kashi, Jiaodong, Beijing, Changchun, Weinan, Handan, Jinan, Qingdao, Taiyuan, Yuanwang-1 (longitude 70°, latitude 0°), Yuanwang-2 (longitude −20°, latitude −20°), Yuanwang-3 (longitude −150°, latitude 0°). The velocity increment offered by the two servicers is $\{2,000\,\mathrm{m/s} \quad 1,900\,\mathrm{m/s}\}$, the initial moment is 2010-01-01 01:00:00 UTC, and the instruction injection lasts for 30 s, i.e., $\Delta t_{\text{ins}} = 30$ s.

Assume the mission priority of the four customers as $\{0.7 \ 0.6 \ 0.9 \ 0.7\}$. The previous algorithm yields the results as shown in Table 2.2.

From Table 2.2, the optimized assignment scheme is Servicer-1 serving Customer-3, and Servicer-2 serving Customer-4. In the former, the servicer stays at the initial orbit for 800 s and then maneuvers; in the latter, the servicer stays at the initial orbit for 3,650 s and then maneuvers. Simulation results show that the derived scheme can offer a feasible optimized solution to address the mission assignment problem.

2.3 HGABB for One-to-N Spacecraft Mission Planning

The one-to-N spacecraft mission planning problem is primarily concerned with sequence arrangement and time distribution. The former aims to seek the best serving sequence and the latter seeks the time distribution that is assigned to each mission, such that the cost function is minimized. The basic solution procedures could be described as below.

Step 1: Select an initial mission sequence.
Step 2: Optimize the time distribution for this mission sequence.
Step 3: Produce a new mission sequence by an optimization algorithm.
Step 4: Iterate Steps 2 and 3 until an acceptable mission sequence and corresponding time distribution are found.

In this sense, one-to-N spacecraft mission planning is a two-level optimization problem, with the leader representing sequence arrangement and the subordinate representing time distribution. Given the satellite density in GEO orbit, the one-to-N approach helps save cost and boost efficiency. In the following section, the analysis is carried out against the inspection scenario over multiple GEO satellites [4, 5], as shown in Fig. 2.3; then the one-to-N servicing mission could be implemented by coplanar phase adjustment of the servicer.

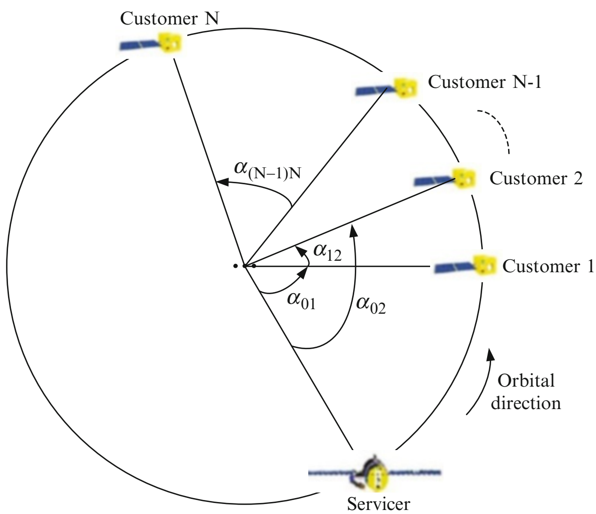

Fig. 2.3 Schematic of one-to-N servicing mission

2.3.1 Planning Model

According to Sect. 2.1.1 and the second assumption in Sect. 2.1.2, the one-to-N spacecraft mission planning model could be simplified as a two-level planning model

$$\begin{aligned} &(P1)\ \min J_2(\mathbf{Q},\mathbf{T}) = \sum_j \sum_i \Delta V_{ij} q_{ij} \\ &\text{s.t.} \\ &\quad \sum_i q_{ij} = 1 \\ &\quad \sum_j q_{ij} = 1 \\ &\quad \sum_j \sum_i \Delta V_{ij} q_{ij} \le \Delta V^{\max} \end{aligned} \tag{2.19}$$

where $i, j \in [1, \ldots, N]$ are the indices of customers of two adjacent missions, and q_{ij} the element of $\mathbf{Q}$ which satisfy

$$q_{ij} = \begin{cases} 1 & \text{maneuver to the } j^{\text{th}} \text{ customer after serving the } i^{\text{th}} \\ 0 & \text{otherwise} \end{cases} \tag{2.20}$$

and ΔV_{ij} is the velocity increment needed to maneuver from the ith to jth customer which satisfies $\Delta V_{ii} = 0$ and $\Delta V_{ij} \ne \Delta V_{ji}$. ΔV_{ij} depends on α_{ij} and $\Delta t_{ij}^{\mathbf{Q}}$ and could be solved by the two-impulse multiple-revolution Lambert transfer algorithm (see Sect. 3.1.1), and $\Delta t_{ij}^{\mathbf{Q}}$ is the time spent by the servicer to maneuver from the ith to jth customer in a mission sequence $\mathbf{Q}$.

The decision variable vector $\mathbf{T}$ could be solved from the lower-level model which is derived to optimize the time distribution, written as

$$\begin{aligned} &(P2)\ \min J_2(\mathbf{T}) = \sum_j \sum_i \Delta V_{ij}\left(\alpha_{ij},\ \Delta t_{ij}^{\mathbf{Q}}\right) \\ &\text{s.t.} \sum_j \sum_i \Delta V_{ij}\left(\alpha_{ij}, \Delta t_{ij}^{\mathbf{Q}}\right) \le\ \Delta V^{\max} \\ &\quad \sum_j \sum_i \Delta t_{ij}^{\mathbf{Q}} \le \Delta T^{\max} \\ &\quad \Delta V_{ij}\left(\alpha_{ij},\ \Delta t_{ij}^{\mathbf{Q}}\right) \le \kappa \Delta V^{\max} \\ &\quad \Delta t_{ij}^{\mathbf{Q}} \le \rho \Delta T^{\max} \end{aligned} \tag{2.21}$$

where κ and $\rho (0 < \rho, \kappa < 1)$ are proportional coefficients, which pose restrictions on reasonable cost. And, $\alpha_{ij} \in [-\pi, \pi]$ is the phase difference between the ith to jth GEO satellite.

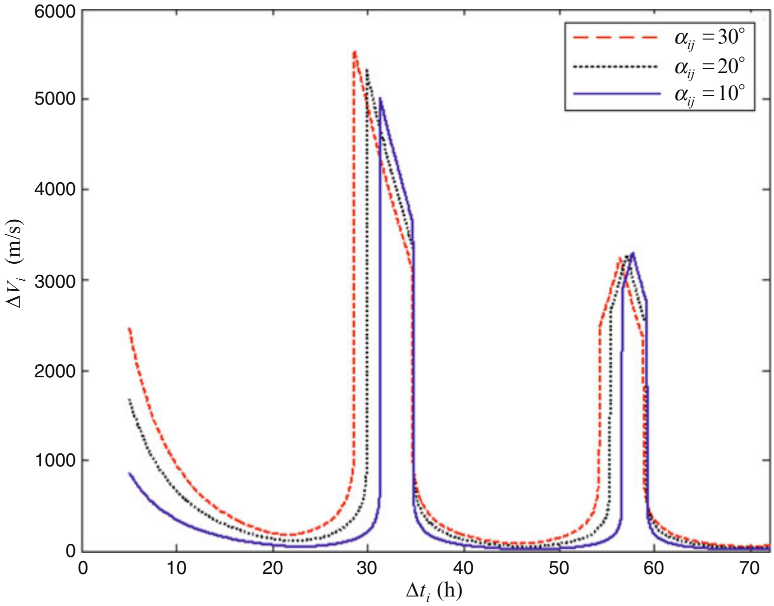

Fig. 2.4 Relationship between Δt_i and ΔV_i at GEO

2.3.2 *Algorithms*

The key to solving Eq. (2.23) is the dynamics property, i.e., the relationship between one certain mission duration Δt_i and velocity increment ΔV_i, which can be addressed by fixed-time two-impulse multiple-revolution Lambert solution (see Sect. 3.1.1) [1]. For a GEO scenario, consider the rendezvous of two co-orbital spacecraft while the maneuvering moment is chosen at the initial time. The result is shown in Fig. 2.4, and we can conclude that ΔV_{ij} can be optimized by adjusting the first impulse moment.

The optimized relationship is shown in Fig. 2.5 which indicates that ΔV_i keeps decreasing with Δt_i. The curves can be viewed as comprising stabilizing and descending segments, which occur in turn. The interval of stabilizing segment, which is usually long, depends on the interval between two local minimum values in Fig. 2.4. Except for the first descending segment, others are of short time. And in descending segments, the velocity increment is time-sensitive. Therefore, simplifications are made by limiting Δt_i in the stabilizing segment. In this way, the problem of time distribution is summarized as how to choose the Δt_i-located segment and determine its specific value.

Define s_i^{num} as the number of the stabilizing segment in Δt_i and $\Delta V_{is}(s = 1, 2, \cdots, s_i^{\text{num}})$ as the velocity increment in the sth stabilizing segment. Further, define t_{is}^{start} as the start moment of the sth stabilizing segment and t_{is}^{end} as its end moment

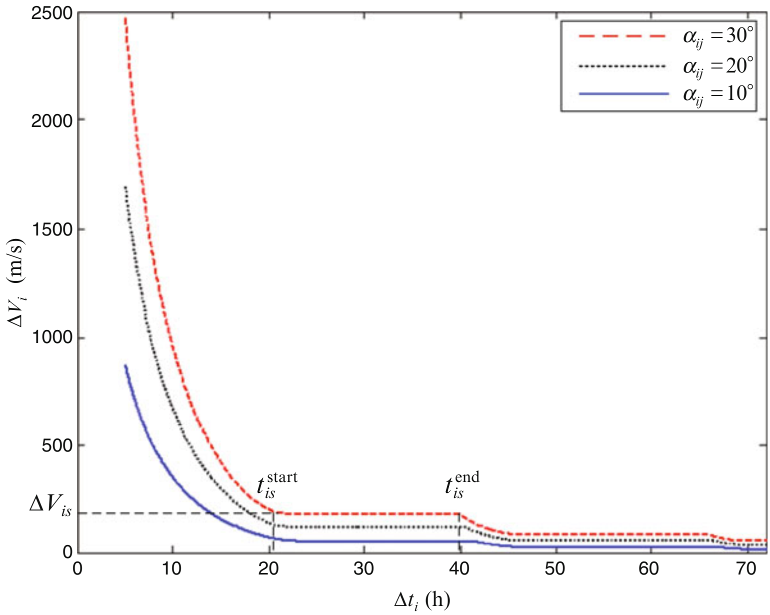

Fig. 2.5 Optimized relationship between Δt_i and ΔV_i at GEO

with respect to the ith mission initial moment. In this way, the lower planning model is transformed to a 0–1 integer programming problem, written as

$$\begin{aligned}
&\min \sum_{i=1}^{N} \sum_{s=1}^{s_i^{num}} \Delta V_{is} y_{is} \\
&\text{s.t.} \quad y_{is} = \begin{cases} 1 & \Delta t_i \in \left[t_{is}^{start}, t_{is}^{end}\right], j = 1, 2, \ldots, s_i^{num} \\ 0 & \text{otherwise} \end{cases} \\
&\qquad \sum_{s=1}^{s_i^{num}} y_{is} = 1 \\
&\qquad \sum_{i=1}^{N} \sum_{s=1}^{s_i^{num}} t_{is}^{end} y_{is} \geq \Delta T^{max} \\
&\qquad \sum_{i=1}^{N} \sum_{s=1}^{s_i^{num}} t_{is}^{start} y_{is} \leq \Delta T^{max} \\
&\qquad \sum_{i=1}^{N} \sum_{s=1}^{s_i^{num}} \Delta V_{is} y_{is} \leq \Delta V^{max}
\end{aligned} \tag{2.22}$$

The first step to solve this model is to calculate t_{is}^{start}, t_{is}^{end}, and ΔV_{is} by the two-impulse multiple-revolution Lambert solution. Then, a 0–1 integer programming

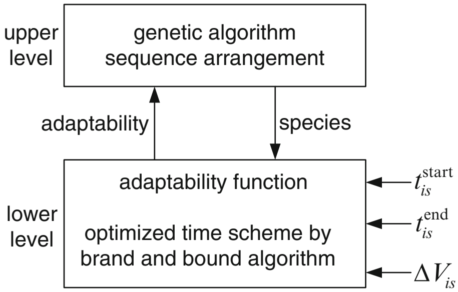

Fig. 2.6 HGABB algorithm

Table 2.3 Simulation results of case 1

Planning algorithm	Enumeration	HGABB
Optimized sequence arrangement	3-2-1-4-5-6	3-2-1-4-5-6
Optimized time distribution (h)	[23.81, 47.92, 23.81, 71.14, 23.61, 47.31]	[23.81, 47.92, 23.81, 71.14, 23.61, 47.31]
Velocity increment (m/s)	68.61	68.61

problem can be solved by common methods of enumeration or branch and bound, the computation amount of which is highly dependent on Δt_i and N. If N is small, enumeration method is better due to its capability of generating a global optimal solution. ΔV_{is} is determined by the Δt_i-located segment and Δt_i can be at any point in the segment. If N is big, the enumeration method is not feasible. Instead, we use the genetic algorithm [6].

The algorithm of this two-level nonlinear planning model is given in Fig. 2.6.

2.3.3 *Numerical Simulation*

Case 1: Take a six GEO satellites inspection mission for example. Assume the mission duration is 240 h, and the initial phase relationship is expressed as

$$[\alpha_{01}, \alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{45}, \alpha_{56}] = \left[-10^{\circ}, 1^{\circ}, 3^{\circ}, 5^{\circ}, 2^{\circ}, 7^{\circ}\right] \tag{2.23}$$

Simulation results are derived by enumeration and HGABB respectively, given as in Table 2.3, which verify the effectiveness of the latter.

Case 2: Take a ten GEO satellites inspection mission for example. Assume the mission duration is 360 h and 720 h respectively, and the initial phase relationship is expressed as

$$\begin{aligned}&\begin{bmatrix}\alpha_{01} & \alpha_{12} & \alpha_{23} & \alpha_{34} & \alpha_{45} & \alpha_{56} & \alpha_{67} & \alpha_{78} & \alpha_{89} & \alpha_{910}\end{bmatrix}\\ &\quad = \begin{bmatrix}37^{\circ} & 10^{\circ} & 13^{\circ} & 4^{\circ} & 32^{\circ} & 5^{\circ} & 6^{\circ} & 12^{\circ} & 25^{\circ} & 7^{\circ}\end{bmatrix}\end{aligned} \tag{2.24}$$

Table 2.4 Simulation results of case 2

Mission duration	360 h (both-optimal)	720 h (both-optimal)
Mission sequence	4-3-2-1-5-6-7-8-9-10	1-2-3-4-5-6-7-8-9-10
Time distribution (h)	24.42, 24.00, 48.64, 24.42, 67.89, 23.39, 23.33, 46.97, 46.11, 23.25	48.44, 72.00, 48.64, 48.44, 67.89, 71.42, 71.33, 70.94, 70.08, 71.28
Velocity increment (m/s)	500.4	320.4
$\sum \lvert\alpha_{ij}\rvert$	151°	151°
$\min \sum \lvert\alpha_{ij}\rvert$	151°	151°

Table 2.5 Comparison of different scenarios

Mission duration	720 h (both-optimal)	720 h (sequence-optimal)	720 h (time-optimal)
Mission sequence	1-2-3-4-5-6-7-8-9-10	1-2-3-4-5-6-7-8-9-10	5-6-7-8-9-10-4-3-2-1 (given)
Time distribution (h)	48.44, 72.00, 48.64, 48.44, 67.89, 71.42, 71.33, 70.94, 70.08, 71.28	Mean	70.28, 71.42, 71.33, 70.94, 70.08, 71.28, 53.58, 72.00, 48.64, 48.44
Velocity increment (m/s)	320.4	445.4	441.7
$\sum \lvert\alpha_{ij}\rvert$	151°	151°	182°
$\min \sum \lvert\alpha_{ij}\rvert$	151°	151°	151°

HGABB generates the planning results as given in Table 2.4. A comparison reveals that mission sequence arrangement and fuel consumption differ as mission duration changes. In Table 2.4, $\min \sum |\alpha_{ij}|$ is the minimum value of $\sum |\alpha_{ij}|$ in all feasible sequences. Simulation results show that $\sum |\alpha_{ij}|$ and $\min \sum |\alpha_{ij}|$ share the same value, indicating that $\sum |\alpha_{ij}|$ in the optimal sequence usually takes the minimum value, and phase relationship is key to fuel consumption. Table 2.5 lists the results of both-optimal, sequence-optimal, and time-optimal scenarios with 720 h mission duration. Simulation results show that fuel consumption is greatly reduced in both-optimal scenario.

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