In this chapter, the important common elements of all lasers are introduced. Some examples of lasing systems are given to define how these elements are implemented in practice.

2.1 Introduction

Semiconductor lasers are the enabling light source of choice for optical communications. However, the basic principles of operation of semiconductor lasers are shared by all lasers. In this chapter, the requirements for lasing systems and the characteristics of all lasers will be discussed. Specific examples from outside semiconductor lasers will be used to demonstrate these characteristics, before we focus on the specific mechanics and structure of semiconductor lasers.

2.2 Introduction to Lasers

With an appreciation of the significance and underlying technology of optical communication, we can start to understand the basic process of lasing. In this section, we introduce the fundamental underpinnings of lasing, stimulated emission. Stimulated emission is the idea that under certain conditions a photon can create additional photons of the same wavelength and phase. Lasers are based on this principle and create “floods” of photons of the same wavelength and phase that constitute laser light.

To start to understand stimulated emission, we begin with a description of one of the classical problems of physics—black body radiation.
2.2.1 Black Body Radiation

Black body radiation is the spectrum emitted from a “black body” (an object without any particular color) as it is heated up. “Red hot” iron and “yellow hot” iron are red and yellow because, at the temperature to which they are heated, their emission peak is $\sim 600$ or $\sim 550$ nm, and they look “red” or “yellow.” The surface of the sun is another example of a classical black body. Measurements showed that black bodies emit light at a peak spectral wavelength depending on their temperature, with the amount of emission above and below that wavelength falling off to zero at shorter and longer wavelengths. The peak emission shifted to shorter wavelengths as the temperature of the black body increased. All black bodies at the same temperature emit light of the same spectrum, independent of the material.

In the beginning of the twentieth century, the physics behind the spectrum was a great mystery to early twentieth century physicists. The shape of the curve was well described by a simple equation first derived by Max Planck,

$$ E(v)dv = \frac{8\pi h v^3}{c^3} \frac{1}{\exp(hv/kT) - 1} dv $$ (2.1)

where $E(v)$ is the amount of energy density, in $J/m^3/Hz$, in each frequency. The theory behind this equation was not understood until quantum mechanics was introduced.

Aside: It is remarkable how powerful and universal this black body spectrum is. Radiation from outer space is difficult to measure on Earth, because the atmosphere absorbs very long wavelengths. The cosmic background explorer (COBE) satellite was sent up to measure the far infrared black body spectrum above the atmosphere. Shown in Fig. 2.1 is one of the spectra it recorded. The shape fits perfectly to the shape of the spectrum of Eq. 2.1, and from this data, the temperature of the universe could be extracted. It turns out that the universe as a whole is a balmy $2.75$ K. This measurement is currently being interpreted as support for the Big Bang theory of the creation of the universe. It was clear that this measurable phenomenon was driven by basic physics. The initial theory and discovery of this cosmic background radiation resulted in Nobel Prizes for Penzias and Wilson in 1978; the subsequent measurements by the COBE satellite resulted in Nobel Prizes for Smoot and Mather.

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This black body formula can be understood in fundamentally two different ways; (i) a macroscopic, statistical thermodynamics viewpoint, attributed to Planck and (ii) a microscope rate equation viewpoint, attributed to Einstein. Both views are correct and both have parallels with semiconductor lasers. The statistical view, involving density of states, is repeated when calculating gain in a semiconductor laser. The rate equation view comes up again when talking about modeling laser DC and dynamic performance. Let us talk about both views in detail.

2.2.2 Statistical Thermodynamics Viewpoint of Black Body Radiation

The viewpoint of statistical thermodynamics, which is fundamentally Planck’s view, is that an existing “state” has a certain probability to be occupied, based on its temperature. As the temperature increases, it becomes more likely that higher energy states are occupied. At a temperature of absolute zero, only the very lowest energy states are occupied; at higher temperatures, the higher level energy states start to be occupied.

As such, the spectrum is determined by two things: first, the probability distribution function, which determines the likelihood that a state will be occupied based on temperature; second, the density of states, which is the number of states that exists at a particular energy in a black body. We will talk about both of these terms in the following sections.

Fig. 2.1 One of the first measurements of the COBE background microwave satellite, showing the use of the optical spectrum of the black body to measure temperature. Image from http://en.wikipedia.org/wiki/File:Cmb.png, current 1/2013
2.2.3 Some Probability Distribution Functions

Let us briefly review probability distribution functions for photons and electrons. A distribution function gives the probability that an existing state will be occupied based on the energy of the state and the temperature of the system. These functions are thermodynamic functions that are applicable to systems in thermal equilibrium at a fixed temperature. Table 2.1 shows a list of the statistical distribution functions and the systems (or particles) to which they apply.

In these functions, \( E \) refers to the energy of the state, \( E_f \) is a characteristic energy of the system (the Fermi energy) usually used with Fermi–Dirac statistics, and \( kT \) is the Boltzmann constant times the temperature (in Kelvin). The constant \( A \) in the Bose–Einstein and Maxwell–Boltzmann functions depends on the type of particles but is 1 for photons.

Example: If the Fermi energy of a semiconductor is 1 eV above the valence band, at room temperature, what is the probability that an electronic state 2 eV above the valence band will be occupied?

Solution: The Fermi–Dirac function applies here, but in fact, \( E - E_f \) is high enough that all three functions will give the same answer:

\[
\exp\left(\frac{1}{C0\cdot0.026\text{eV}}\right) = \exp\left(\frac{-40}{18}\right) = 10^{-18}.
\]

The Bose–Einstein distribution function is appropriate for photons, phonons, and particles with integral spin (like protons) and reflects the fact that these particles can have any number of particles in a given state.

The Fermi–Dirac function applies to particles which follow the Pauli exclusion principle that at most one particle can occupy a given energy state. Let us take this very earliest opportunity to note that this exclusion principle excludes more than one particle from each quantum state, not from each energy level. A quantum state is a set of quantum numbers that describe a particle. Many situations have multiple states with the same energy that have different sets of quantum numbers, such as the sublevels of \( p \)-orbital of an atom. These states are called degenerate in energy.

This distribution function is only a part of the story. The population of electrons present at any given energy depends on the number of states at that energy. The bandgaps of semiconductors are devoid of states, because of their particular

| Table 2.1 Distribution Functions \( P(E) \) \( dE \) |
|-----------------|-------------------|-------------------|
| Distribution function name | Function | Applies to |
| Bose–Einstein | \( \frac{1}{A \exp(E/kT) - 1} \) | Bosons: photons and protons and spin \(-1\) particles |
| Fermi–Dirac | \( \frac{1}{\exp((E - E_f)/kT) + 1} \) | Electrons and other spin \( \frac{1}{2} \) particles |
| Maxwell–Boltzmann | \( A \exp\left(-\frac{E}{kT}\right) \) | All particles at high temperatures |
crystalline arrangement. In order to determine the population of photons, we have to derive the density of states, or the number of photon states that are available to be occupied at any given energy.

### 2.2.4 Density of States

In order to apply the distribution functions, a state must exist. These states are allowed solutions of the Schrödinger Equation for a particular physical situation or potential.

The calculation of the density of states in black body is best illustrated by an example. Let us proceed to consider the density of photon states for a cubic black body with length \( L \) per side, and calculate what the density of states per unit energy \( D(E) \, dE \) is. A picture of a cubic black body volume is shown in Fig. 2.2. The “volume” is considered to be macroscopic and much larger than the wavelength of the photons corresponding to this energy.

An intuitive picture suggests that for a given volume, there should be many more short wavelength, high energy photons, per volume than long wavelength, low-energy photons.

The conventional approach here is to pick an electromagnetic boundary condition that confines photons within the black body, and allow only wavelengths that are integral fractions of the cubic length \( L \). For example, wavelengths of \( \lambda_x = L \) are allowed, and wavelengths \( \lambda_x = L/2 \) are allowed, but a wavelength of \( \lambda_x = 0.8L \) is not allowed. The same applies to wavelengths in the other two directions, \( \lambda_y \) and \( \lambda_z \) (Fig. 2.2).

Let us calculate the number of these allowed photon states that exist as a function of energy in a black body.

It is easier to analyze this problem in what is called *reciprocal space*, in which the propagation constants \( k \) rather than the wavelengths are considered. If the wavelength is \( \lambda_x \), the propagation constant \( k_x = 2\pi/\lambda_x \). This relationship is true for wavelengths of the components of the photon in each of the three directions, as well as the scalar wavelength of the photon and the amplitude of \( k \).

We are going to write the relationship between \( \lambda \) and \( k \) in two ways (shown below); the first between the vector \( x \), \( y \), and \( z \) components of \( k \), and the second

![Fig. 2.2 A cubic black body of macroscopic size](image)
between the magnitude of \( k \) and magnitude of \( \lambda \). The magnitudes of \( k \) and \( \lambda \) are related to their magnitudes in the three orthogonal directions as shown.

\[
k_{x,y,z} = \frac{2\pi}{\lambda_{x,y,z}}
\]

\[
k = \sqrt{k_x^2 + k_y^2 + k_z^2}
\]

\[
1 = \sqrt{\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} + \frac{1}{\lambda_z^2}}
\]

\[
\lambda = \frac{2\pi}{k}
\]

The simplest way to understand the propagation constants is to consider them as reciprocals of the wavelength \( \lambda \). The product of wavelength and propagation constants is a full cycle, \( 2\pi \). If the wavelength halves, the propagation constant doubles. Writing the allowed wavelengths and propagation constants in terms of the boundary conditions above gives a picture of the spacing of the allowed propagation constants.

The allowed wavelengths are integral fractions of the cavity length and so, the allowed propagation constants are integral multiples of the fundamental propagation constant, \( 2\pi/\lambda \) as shown in the expressions below.

\[
\lambda_{\text{allowed}} = \frac{L}{m_{x,y,z}}
\]

\[
k_{\text{allowed}} = \frac{m_{x,y,z}}{L} \frac{2\pi}{\lambda}
\]

These allowed propagation constants form a set of evenly spaced grid points in the reciprocal space plane, as shown below in 2D (\( x \) and \( y \)). Any point represents a valid propagation constant of a photon, and \( k \)-values between the points cannot exist in a black body.

The vector \( k \), having \( k_x \), \( k_y \), and \( k_z \) components, gives the propagation direction, and the quantization condition (Eq. 2.3) is independently fulfilled in each direction.

Figure 2.3 shows the picture of allowed \( k \)-states in \( x \) and \( y \). Using this diagram, and the probability distribution function for photons, we calculate the density of photons at a given frequency (the black body spectrum, Eq. 2.1). What is the number of states at a given energy as a function of the optical frequency \( v \) (\( \text{N} (\nu) \text{ d}\nu \))?

First, we realize that by Plank’s formula, \( E = h\nu \), the optical frequency, or wavelength \( \lambda \) equivalently specify the energy.

\[
E = h\nu = \frac{hc}{\lambda} = \frac{hck}{2\pi} = hck
\]
Even though $k$ is a vector as above, the $k$ in this expression is the scalar magnitude of $k$. In the picture above, anything with same magnitude (shown by the circle) has the same energy. *Calculating the density of states is equivalent to calculating the density of points of a circle of radius $k$.*

The picture above, for clarity, is actually a 2D picture slice of the 3D system. We are going to carry through the derivation in 3D in which there are three dimensions of allowed propagation vectors, in $x$, $y$, and $z$. The procedure we follow is to calculate the differential volume in a thin slab of fixed radius $dk$, then divide by the volume per point to get the number of points in that volume. We find that the differential volume for a 3D segment is

$$V(k)dk = 4\pi k^2 dk$$ (2.5)

The density of points as a function of $k$, $D_p(k)$, is given by this volume divided by the density of states in $k$-space, which is 1 state per $(2\pi/L)^3$ volume, or

$$D_p(k)dk = \frac{4\pi k^2 dk}{(\frac{2\pi}{L})^3} = \frac{L^3 k^2}{2\pi^2} dk$$ (2.6)

Finally, the relationship between energy and $k$ is best expressed as follows: (and substituted into the above)

$$E = \hbar ck \quad dE = \hbar c \; dk$$

$$k = \frac{E}{\hbar c} \quad dk = \frac{dE}{\hbar c}$$ (2.7)
Substituting into the above expression, we obtain

\[
D_p(E)dE = \frac{4\pi E^2 dE}{\hbar^3 c^3 (\frac{2E}{\hbar})^3} = \frac{L^3 E^2 dE}{2\pi^2 \hbar^3 c^3} \tag{2.8}
\]

Considering the density of states per fixed real space volume, \(L^3\), gives us the nearly final result for the density of points in \(k\)-space \((D_p)\) equal to,

\[
D_p(E)dE = \frac{E^2 dE}{2\pi^2 \hbar^3 c^3} \text{ cm}^{-3} \tag{2.9}
\]

A final factor of two has to be multiplied to the expression above to give the density of photon states. Each state, in addition to direction, has a polarization. The polarization can be uniquely specified with two orthogonal polarization states, and as a result the density of state is doubled and the final expression for total density of states, \(D(E)\), is

\[
D(E)dE = \frac{E^2 dE}{\pi^2 \hbar^3 c^3} \text{ cm}^{-3} \tag{2.10}
\]

We have derived this equation in such detail because this will echo the discussion of density of states in an atomic solid, and the very same principles will be used to write down a “density of states” for electrons and holes in exotic quantum confined structures, like quantum wells (a 2D slab), quantum wires (a 1D line), or quantum dots (small chunks of material with dimensions comparable to atomic wavelength).

Let us make some comments about this derivation, so far. First, there is a key role about the dimensionality of the solid. The expression for “differential volume” contains \(k^2\), which is what leads to the quadratic dependence of \(D(E)\) on \(E\). When we start discussing atomic solids, particularly 2D quantum wells (QWs), 1D quantum wires, and 0-D quantum dots (QDs), this dimensionality will be different and the density of states will have a different dependence on energy.

Second, let us emphasize again what the term “density of states” means. It means only the number of states with the same energy, but not with the same quantum numbers. In a black body, for example, there are red photons radiating in all directions, with different quantum numbers \(k_{x,y,z}\) but the same wavelength (energy). Density of states measures the number of photons with that red energy or wavelength.

Third, looking back, there is a key assumption about the electromagnetic boundary condition perfectly confining the photons, which is only reasonable and not rigorous.
2.2.5 Spectrum of a Black Body

Having discussed density of states and calculated the density of states in a black body, we now talk about the spectrum of a black body. The statistical thermodynamics way of looking at it is simple: multiply the density of states by the distribution function (giving the probability that the existing state is occupied) to determine the occupation or emission spectrum. In this case, written as a function of energy, the number of photons \( N(E) \) at that energy is:

\[
N(E) \, dE = \frac{1}{\exp(E/kT) - 1} \frac{E^2 \, dE}{\pi^2 h^3 c^3} \, \text{cm}^{-3}
\]  

(2.11)

Or as a function of energy \( \rho(E) \) (energy/cm\(^3\)), it simply gets multiplied by another \( E \) to obtain

\[
\rho(E) \, dE = \frac{1}{\exp(E/kT) - 1} \frac{E^3 \, dE}{\pi^2 h^5 c^3} \, \text{cm}^{-3}
\]  

(2.12)

It is left as an exercise to the student to substitute back in \( E = \hbar \nu \) and obtain Planck’s black body spectra, Eq. 2.1.

All of this discussion should be relatively familiar. We now want to look at this problem in a slightly different way and see what insights we can get in particular about lasing.

2.3 Black Body Radiation: Einstein’s View

The preceding discussion about black bodies introduced (or reminded) the reader of distribution functions, and density of states, and both of these concepts will reappear again in the context of semiconductor lasers. However, let us consider a microscope rate equation view, attributed to Einstein, which considers the processes that the photons undergo to maintain that distribution.

Let us consider for a moment, the “sea” of electrons and atoms in a metal which constitute a black body. At any given moment, some number of photons are being absorbed by the metal with the electrons rising to a higher energy level, and some other photons are being emitted as the electrons relax to a lower energy level. For a black body (which is a temperature-controlled, thermodynamic system) at a fixed temperature, these rates of up and down transitions have to be the same for the black body to be in equilibrium. The rate of photons being absorbed has to equal the rate of photons being emitted.

What Einstein postulated was three separate processes which go on in a black body:

1. Absorption in which a photon is absorbed by the material and the material (or electron in the material) is left in an excited state.
(2) Spontaneous emission, in which the material or photon relaxes to a lower energy state and a photon is emitted, without the influence of another photon.

(3) Stimulated emission, in which the material or electron relaxes to another energy state and a photon is emitted, when stimulated by another photon.

These three processes are illustrated in Fig. 2.4. It is this last process which is the process responsible for lasing and which we will discuss in much detail. It is likely to be unfamiliar to the student. The proof that in fact it is a valid physics process, as valid as gravitation, will be found in the equivalence of this model with the statistical thermodynamic model of black body emission, when this mechanism is considered.

Let us now proceed to establish the correspondence between these two models. In equilibrium, the rates of the excitation and relaxation processes must be equal. Let us go ahead and postulate the following linear model for the relative rates.

The processes pictured in Fig. 2.5 can be written down conceptually, in equilibrium, as

$$AN_2 + B_{21}N_2N_p(E) = B_{12}N_1N_p(E)$$

(2.13)

where $N_2$ and $N_1$ are the fraction of the populations in the states $N_2$ with energy $E_2$ and $N_1$ with energy $E_1$, respectively; $N_p(E)$ is the photon density as a function of energy.
energy $E = E_2 - E_1$. $A$ is a linear proportionality coefficient for the rate of absorption, and $B_{12}$ and $B_{21}$ are the linear coefficients for the rates of stimulated emission and absorption, respectively. We include one more physical fact, that the populations in state $N_1$ and state $N_2$ are in thermodynamic equilibrium, as

$$\frac{N_2}{N_1} = \exp\left(-\frac{(E_2 - E_1)}{kT}\right)$$

(2.14)

with $E_2$ and $E_1$ the energy of the states. With these facts, it is possible to show that the black body spectra, $N_p(E)$ is the same as that derived earlier if the two Einstein $B$ coefficients for stimulated emission and absorption are equal (and we will henceforth write them just as $B$). This will be left as an exercise for the student (see Problem P2.2)!

### 2.4 Implications for Lasing

The sense of lasing is of a monochromatic and in-phase beam of light. The process of stimulated emission is one in which a single photon stimulates the emission of another photon, which stimulates additional photons (still in phase at the same wavelength) leading to an avalanche of identical photons. The mechanism which does this is stimulated emission; therefore, what is desired is a physical situation in which the rate of stimulated emission is greater than the rate of absorption or of spontaneous emission. The word laser, which is now accepted as a noun, was originally an acronym for Light Amplification by Stimulated Emission of Radiation.

The reader can observe that the rate equation appears from nowhere and has no justification, but stipulates a new process (stimulated emission) which is nontrivial. This is true, but this has proven, over time to be an accurate model of the world, and so it has been retained. We take the equation above as valid and will examine it for the implications it has for lasing.

Let us now make some observations about the equation above and see what it indicates about a lasing system.

First, it describes dynamic equilibrium. In the material, electrons are constantly absorbing and emitting photons, but the population of excited and ground state electrons and photons stays constant. The units of each of the terms on each side of the equation are rates ($/\text{cm}^3\cdot\text{s}$). When these transition rates are equal, the equation describes a steady state situation; in thermal equilibrium, the populations can be described by a Boltzmann distribution and the relative size of the populations are as given in Eq. 2.14.

In equilibrium, the population of the higher energy state is always lower than that of the lower energy state, and therefore the rate of absorption is always greater than the rate of stimulated emission:$BN_2N_p(E) > BN_1N_p(E)$ (the absorption rate is
always greater than the stimulated emission rate in thermal equilibrium). Not only is the absorption rate greater, but enormously greater. In a typical semiconductor laser, $E_2 - E_1 \sim 1$ eV, which gives the relative population of ground and excited states as $\exp(-40)$ at room temperature. Because in equilibrium $N_2 \ll N_1$, stimulated emission is much less than absorption, and therefore in equilibrium lasing is not possible.

This means practical lasing systems must be driven in some nonequilibrium way, generally either optically or electrically. It is not possible to drive something thermally and achieve a dominant stimulated emission. Practical lasing systems are usually composed of (at least) three levels: an upper and lower level, between which the system relaxes and emits light, and a third, pump level, where the system can be excited. This will be illustrated in Sect. 2.6.

In addition, for lasing to occur, the spontaneous emission rate must also be much less than the stimulated emission rate. While both processes produce photons, the spontaneous emission photons are emitted at random times and are thus in random phases compared to the coherent photons generated by stimulated emission. These photons thus do not really contribute to the coherent lasing photons. For a lasing system, $BN_2N_p(E) > AN_2$.

This may or may not be possible depending on the relative values of $A$ and $B$ and various $N_s$. We note that a higher photon density, $N_p$, certainly makes the balance favorable. There is much more stimulated emission at higher photon density than at lower photon density. Hence, for stimulated emission to dominate, it is beneficial to have a higher photon density. This is achieved in a laser by always having some cavity mechanism, based on mirrors or other wavelength-selective reflectors, to achieve a high photon density inside the cavity.

The first equation (stimulated emission greater than absorption) implies that the lasing system is nonequilibrium ($N_2 > N_1$) and is called population inversion. The second equation (stimulated emission greater than spontaneous emission) implies a high photon density. These two conditions taken together form a mathematical model for a physical basis for a lasing system.

\[
BN_2N_p(E) > AN_2 \quad \Rightarrow \quad \text{high photon density } N_p
\]

\[
BN_2N_p(E) > BN_1N_p(E) \quad \Rightarrow \quad \text{nonequilibrium system with } N_1 < N_2
\]

The first condition means that we cannot construct a laser that will just heat up and lase. Any heat-driven process is by definition a thermal equilibrium process, and in such processes absorption, rather than emission, will always dominate. This nonequilibrium requirement is realized in real laser systems by having them powered—for example, in semiconductor lasers, the holes and electrons are electrically injected rather than thermally created. These requirements are illustrated in Fig. 2.6. The portion of a lasing system which is in population inversion is called the gain medium.
In the next two sections, we are going to talk about the qualitative differences between spontaneous emission, stimulated emission and lasing, and give some examples about how these two requirements for lasing systems (nonequilibrium excitation and high photon density) are implemented in practice.

### 2.5 Differences Between Spontaneous Emission, Stimulated Emission, and Lasing

Figure 2.7 illustrates the spectra of some systems dominated by lasing, spontaneous and stimulated emission, to give some intuition to the idea of lasing as a beam of coherent photons and some idea of what is meant by lasing. There is no clean mathematical definition of lasing; the sense of lasing is a monochromatic beam of photons that is dominated by stimulated emission. Figure 2.7 shows the spectrum for a standard semiconductor laser (a distributed feedback laser) whose spectra is dominated by stimulated emission which shows a near-monochromatic one wavelength peak; the spectrum of a light-emitting diode, whose emission...

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**Fig. 2.6** The requirements for a lasing system and the way they are implemented in practice. Nonequilibrium pumping is done electrically, or optically, to excite most of the states. A high photon density is achieved by mirrors or other sorts of optical reflectors to maintain a high photon density inside the cavity. A laser usually looks similar to this conceptual picture.
shows a broad peak characteristic of spontaneous emission from the bandgap of the semiconductor; and finally, a doped Eu system which has achieved population inversion but not an extremely high photon density, and as such exhibits a spectral narrowing but not to the extent seen in (a) We will refer back to this figure and discuss some of the details of the spectra later in this book; for now, we just wish the reader to note that one laser characteristic is an extremely narrow spectra, and that there is a different qualitative character to each of the different mechanisms of stimulated emission, spontaneous emission, and lasing.

In the middle figure, also note that the power density where the system starts to exhibit substantial stimulated emission \( BN_2N_p > AN_2 \) is quite clear. There is also a dynamic element in these lasing systems. Because the population must be inverted \( N_2 > N_1 \), the amount of time an excited state exists before it relaxes is extremely important and can influence properties like the threshold of lasing systems. This also will be talked about in greater detail later.

We note also that absorption can be considered a “stimulated” process, which is the opposite of stimulated emission.

### 2.6 Some Example Laser Systems

All lasers consist of a gain medium, a method of nonequilibrium pumping, and a cavity defined by mirrors or another mechanism to obtain a high photon density. We now show specific examples illustrating how these three properties are achieved. Because the bulk of the book will discuss semiconductor lasers, these examples are going to be taken from other laser systems.

Apart from the gain medium, this will also show the various ways in which optical cavities are formed to contain the photons.
First, an Er-doped fiber laser has the atomic levels of the erbium (Er) atom as the gain medium, optical pumping as the means for inducing nonequilibrium, and a Bragg grating cavity integrated into the fiber as the cavity mirror to achieve a high photon density.

Second, we will talk about a common red He–Ne gas laser, which has the Ne atomic levels as the gain medium, a high-voltage AC source as the method of electrically exciting (pumping) the molecules, and high reflectivity mirrors defining the cavity.

### 2.6.1 Erbium-Doped Fiber Laser

As an illustration, Fig. 2.8 depicts the energy levels and the physical structure of an erbium (Er)-doped fiber laser. This structure is similar to an Er-doped fiber amplifier, but with an engineered cavity. An optical fiber is fabricated doped with optically active Er atoms, and a simplified version of the Er atomic energy level is shown at above left. Pump light at 1 μm excites the atoms into an excited state (the 4I_{15/2} state), which then rapidly (∼ns) relaxes into a state with a band gap at around 1 μm (the 4I_{13/2} state). This state has a lifetime of ∼ms, and so the system can be put into population inversion in which the density of atoms in the 4I_{11/2} state is much higher than the 4I_{13/2} state. Here, the three states (4I_{15/2}, 4I_{13/2}, and 4I_{11/2}) are the pump level, upper level, and lower level, respectively.

The dynamics are actually critical to this system. If the relaxation between 4I_{15/2} and 4I_{13/2} were slower, or the relaxation between 4I_{13/2} and 4I_{11/2} were faster, it would be much harder to achieve “population inversion” system in which the population of 4I_{13/2} > 4I_{11/2}, as required for lasing.

The other requirement for lasing is high photon density. This is accomplished by the Bragg gratings integrated into the fibers, which confine most of the 1.55 μm photons into the fiber laser cavity. In order to allow the pump light in freely, these gratings have to have a low reflectivity at 1 μm. This system produces a device which, when high-intensity 1 μm light is coupled into the fiber, produces a monochromatic beam of 1.55 μm light out.

### 2.6.2 He–Ne Gas Laser

The traditional red laser that is often used in optics laboratories is a He–Ne gas laser. The schematic picture of such a lasers and its mechanism for operation is shown in Fig. 2.9. The gain medium is the He–Ne molecules that are sealed in the tube. A high DC voltage is applied which creates electrons which excite a He atom. The He atom then transfers its energy to a Ne atom. The Ne atom then relaxes by radiative stimulated emission to a lower level, emitting a red photon at λ = 632 nm in the process. Even though the light has already been emitted, the Ne atom then has to relax through several more levels nonradiatively down to the
ground state to be reused. Finally, the photons are kept in the cavity by the mirrors at each end of the tube. The reflectivity is typically ~99% or more, so the photon density inside the laser is much, much higher than the photon density right outside the cavity.

There are several atomic levels to the Ne atom. By tailoring the cavity to confine photons of different wavelengths (a mirror specific to red, green, or infrared wavelengths), the same system can be induced to lase in the green or

---

**Fig. 2.8** An erbium-doped fiber laser. As shown, population inversion is achieved between the $4\text{i}_{13/2}$ and $4\text{i}_{11/2}$ level by optical pumping, a nonequilibrium process. High photon density is achieved by Bragg mirrors, which keep most of the 1.55 μm photons in the laser length of the fiber.
infrared as well as red. Commercial He–Ne lasers at all these wavelengths can be purchased.

In Fig. 2.9, the upper portion shows the atomic level picture of the mechanism for operation of the He–Ne laser. The molecule is initially excited, and the relaxation time from the excited state is long enough that the system can be put into population inversion. Once population inversion is achieved, lasing occurs because stimulated emission dominates and the photon density is kept high with the highly reflective facets. The laser cavity is shown at the bottom.

Semiconductor lasers will be covered extensively in following chapters. In general, they have electrical injection as the pumping method, with the conduction and valence bands serving as the gain medium. There are many mirror methods available in semiconductor lasers; the simplest one is simply the mirror formed when the semiconductor with the refractive index \( n = 3.5 \) is cleaved, and an interface with the air \( (n = 1) \) is formed.

**Fig. 2.9** A He–Ne gas laser, showing the gain medium (the Ne atom), the high photon density (created by high reflectivity mirrors), and the method for nonequilibrium pumping by electronic excitation. The **bottom** shows the physical picture of a He–Ne laser; the tube is the active laser region, while the area around it is a reserve gas cavity.
2.7 Summary and Learning Points

A. Distribution functions describe the probability that an existing energy state is occupied. They describe systems in thermodynamic equilibrium. Different functions are appropriate to different situations. The Fermi–Dirac distribution function is applicable to particles which follow the exclusion principle (electrons or holes); the Bose–Einstein is applicable to photons or protons or other particles who like to aggregate; and the Boltzman distribution function is the classical approximation to both.

B. The density of states function is the number of states at a given energy in a system. The density of photon states in a black body can be calculated and that, combined with the appropriate distribution function, gives the black body emission spectra.

C. By equating the rates of particle relaxation and excitation (in a “dynamic” equilibrium), the same picture of black body emission spectra can be obtained (provided that the two Einstein B coefficients are equal). This model resulted in defining the (new) mechanism of light emission called stimulated emission, in which a photon impinges on an excited atom and causes it to emit another photon of the same wavelength and phase. It is this mechanism that is responsible for lasing.

D. A laser is a coherent light source generated by stimulated emission. Hence, stimulated emission has to dominate over both absorption and spontaneous emission. These criteria require a lasing system to:
   i. be in population inversion, with more of the gain medium in the excited state than in the ground state.
   ii. have a high photon density $N_P$, which requires mirrors or facets to surround the lasing system.

E. Because of the population inversion requirement, a laser cannot be driven thermally. Lasers are nonequilibrium systems.

2.8 Questions

Q 2.1 Define stimulated emission of radiation.
Q 2.2 Explain how the temperature can be measured from a black body spectrum.
Q 2.3 Explain in your own words the statistical thermodynamics perspective of black body radiation.
Q 2.4 Explain in your own words the microscopic view of black body radiation.
Q 2.5 Define the term “distribution function”.
Q 2.6 Define the term “population inversion”.
Q 2.7 What distribution function is appropriate for photons? For electrons?
Q 2.8 When is it appropriate to use the Gaussian distribution function?
Q 2.9 Define the term “density of states”.

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Q 2.10 If the $k$-value of a particular photon state is very large, is the wavelength of that photon high or low? Is the energy of that photon high or low?

Q 2.11 List the three requirements for any lasing system.

Q 2.12 Explain how these requirements are met in your own words for the two types of lasers discussed in the chapter.

Q 2.13 What are the three levels in the He–Ne laser system?

### 2.9 Problems

P 2.1 Show that Eq. 2.11 reduces to Plank’s expression for a black body spectrum, Eq. 2.1.

P 2.2 Show that for a system in thermal equilibrium, the coefficient of stimulated emission $B_{21}$ is equal to the coefficient of stimulated absorption $B_{12}$. (Hint: use the fact that the $N_2/N_1 = \exp(-\Delta E/kT)$, and the fact the Einstein and Plank black body spectra must agree).

P 2.3 A photon has a wavelength of 500 nm.
   (i) What color is it?
   (ii) What is its energy, in?
       (a) J
       (b) eV.
   (iii) What is the magnitude of its spatial propagation vector $k$?
   (iv) Find its frequency in Hz.

P 2.4 (This problem is given by Kasap, and used by permission). Given a 1 μm cubic cavity, with a medium refractive index $n = 1$:
   (a) show that the two lowest frequencies which can exist are 260 and 367 THz.
   (b) Consider a single excited atom in the absence of photons. Let $p_{sp1}$ be the probability that the atom spontaneously emits a photon into the (2,1,1) mode, and $p_{sp2}$ be the probability density that the atom spontaneously emits a photon with frequency of 367 THz. Find $p_{sp2}/p_{sp1}$.

P 2.5 This problem explores the influence of dynamics on the populations of the erbium atom levels. In Figure 2.8, the energy levels of the erbium atom are pictured.
   (a) If a population of Er atoms absorbs $10^{18}$ photons/second, but the lifetime of the excited state is 1ns, what is the steady-state population of atoms in the 4I$_{11/2}$ state?
   (b) If the lifetime of the 4I$_{13/2}$ state is 1mS, what is the steady state population of the 4I$_{13/2}$ state?
   (c) How many 1.55 μm photons are emitted per second?

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