Designing Minimum Guaranteed Return Funds

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Abstract In recent years there has been a significant growth of investment products aimed at attracting investors who are worried about the downside potential of the financial markets. This paper introduces a dynamic stochastic optimization model for the design of such products. The pricing of minimum guarantees as well as the valuation of a portfolio of bonds based on a three-factor term structure model are described in detail. This allows us to accurately price individual bonds, including the zero-coupon bonds used to provide risk management, rather than having to rely on a generalized bond index model.

Keywords Dynamic Stochastic Programming · Asset and Liability Management · Guaranteed Returns · Yield Curve · Economic Factor Model

1 Introduction

In recent years there has been a significant growth of investment products aimed at attracting investors who are worried about the downside potential of the financial markets for pension investments. The main feature of these products is a minimum guaranteed return together with exposure to the upside movements of the market.

There are several different guarantees available in the market. The one most commonly used is the nominal guarantee which guarantees a fixed percentage of the initial investment. However there also exist funds with a guarantee in real terms which is linked to an inflation index. Another distinction can be made between fixed and flexible guarantees, with the fixed guarantee linked to a particular rate and the flexible to for instance a capital market index. Real guarantees are a special case of flexible guarantees. Sometimes the guarantee of a minimum rate of return is even set relative to the performance of other pension funds.

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Return guarantees typically involve hedging or insuring. Hedging involves eliminating the risk by sacrificing some or all of the potential for gain, whereas insuring involves paying an insurance premium to eliminate the risk of losing a large amount.

Many government and private pension schemes consist of defined benefit plans. The task of the pension fund is to guarantee benefit payments to retiring clients by investing part of their current wealth in the financial markets. The responsibility of the pension fund is to hedge the client's risk, while meeting the solvency requirements in such a way that all benefit payments are met. However at present there are significant gaps between fund values, contributions made by employees, and pension obligations to retirees.

One way in which the guarantee can be achieved is by investing in zero-coupon Treasury bonds with a maturity equal to the time horizon of the investment product in question. However using this option foregoes all upside potential. Even though the aim is protect the investor from the downside, a reasonable expectation of returns higher than guaranteed needs to remain.

In this paper we will consider long-term nominal minimum guaranteed return plans with a fixed time horizon. They will be closed end guarantee funds; after the initial contribution there is no possibility of making any contributions during the lifetime of the product. The main focus will be on how to optimally hedge the risks involved in order to avoid having to buy costly insurance.

However this task is not straightforward, as it requires long-term forecasting for all investment classes and dealing with a stochastic liability. *Dynamic stochastic programming* is the technique of choice to solve this kind of problem as such a model will automatically hedge current portfolio allocations against the future uncertainties in asset returns and liabilities over a long horizon (see e.g. Dempster *et al.*, 2003). This will lead to more robust decisions and previews of possible future benefits and problems contrary to, for instance, static portfolio optimization models such as the Markowitz (1959) mean-variance allocation model.

Consiglio *et al.* (2007) have studied fund guarantees over single investment periods and Hertzog *et al.* (2007) treat dynamic problems with a deterministic risk barrier. However a practical method should have the flexibility to take into account multiple time periods, portfolio constraints such as prohibition of short selling and varying degrees of risk aversion. In addition, it should be based on a realistic representation of the dynamics of the relevant factors such as asset prices or returns and should model the changing market dynamics of risk management. All these factors have been carefully addressed here and are explained further in the sequel.

The rest of the chapter is organized as follows. In Section 2 we describe the stochastic optimization framework, which includes the problem set up, model constraints and possible objective functions. Section 3 presents a three-factor term structure model and its application to pricing the bond portfolio and the liability side of the fund on individual scenarios. As our portfolio will mainly consist of bonds, this area has been extensively researched. Section 4 presents several historical backtests to show how the framework would have performed had it been implemented in practice, paying particular attention to the effects of using different objective functions and varying tree structures. Section 5 repeats the backtest when the stock index is modeled as a jumping diffusion so that the corresponding returns exhibit fat tails and Section 6 concludes. Throughout this chapter boldface is used to denote random entities.

2 Stochastic Optimization Framework

In this section we describe the framework for optimizing minimum guaranteed return funds using stochastic optimization. We will focus on risk management as well as strategic asset allocation concerned with allocation across broad asset classes, though we will allow specific maturity bond allocations.

2.1 Set Up

This chapter looks at several methods to optimally allocate assets for a minimum guaranteed return fund using expected average and expected maximum shortfall risk measures relative to the current value of the guarantee. The models will be applied to eight different assets: coupon bonds with maturity equal to 1, 2, 3, 4, 5, 10 and 30 years and an equity index, and the home currency is the euro. Extensions incorporated into these models are the presence of coupon rates directly dependent on the term structure of bond returns and the annual rolling of the coupon-bearing bonds.

We consider a discrete time and space setting. The time interval considered is given by $\left\{0, \frac{1}{12}, \frac{2}{12}, \ldots, T\right\}$, where the times indexed by $t = 0, 1, \ldots, T-1$ correspond to decision times at which the fund will trade and T to the planning horizon at which no decision is made, see Figure 1. We will be looking at a five-year horizon.

Uncertainty Ω is represented by a *scenario tree*, in which each path through the tree corresponds to a *scenario* ω in Ω and each node in the tree corresponds to a time along one or more scenarios. An example scenario tree is given in Figure 2. The probability $p(\omega)$ of scenario ω in Ω is the reciprocal of the total number of scenarios as the scenarios are generated by Monte Carlo simulation and are hence equiprobable.

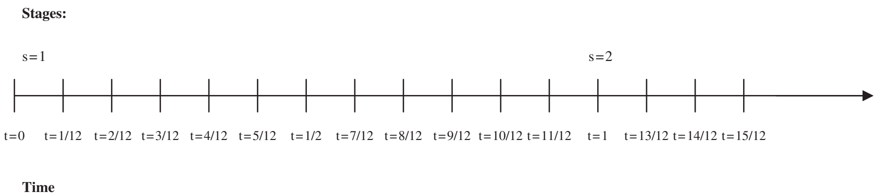

Fig. 1 Time and stage setting

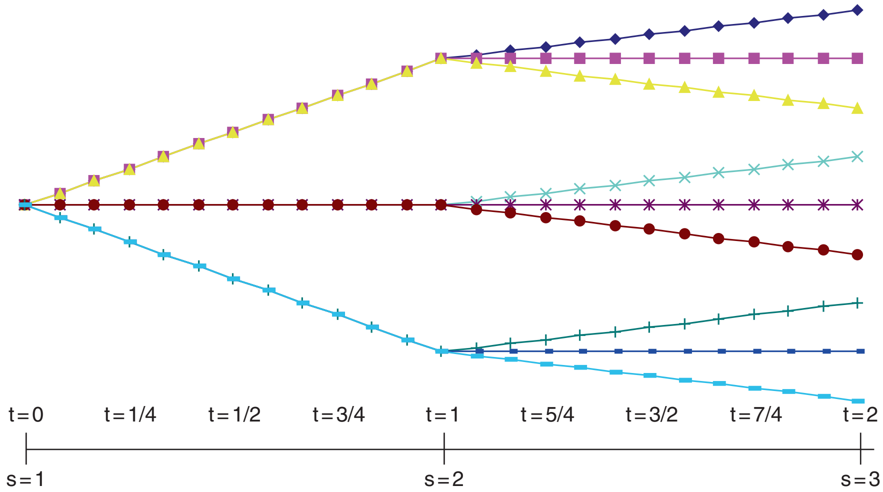

Fig. 2 Graphical representation of scenarios

The stock price process $\mathbf{S}$ is (initially) assumed to follow a geometric Brownian motion, i.e.

$$\frac{d\mathbf{S}_t}{\mathbf{S}_t} = \mu_S dt + \sigma_S d\mathbf{W}_t^S, \tag{1}$$

where $d\mathbf{W}_t^S$ is correlated with the $d\mathbf{W}_t$ terms driving the three term structure factors discussed in Section 3.

2.2 *Model Constraints*

Let (see Table 1)

- $h_t(\omega)$ denote the *shortfall* at time $t$and scenario ω, i.e.

$$h_t(\omega) := \max(0, L_t(\omega) - W_t(\omega)) \quad \forall \omega \in \Omega \quad t \in T^{\text{total}} \tag{2}$$

- $H(\omega) := \max_{t \in T^{\text{total}}} h_t(\omega)$ denote the *maximum shortfall* over time for scenario ω.

The constraints considered for the minimum guaranteed return problem are:

- *cash balance constraints*. These constraints ensure that the net cash flow at each time and at each scenario is equal to zero

$$\sum_{a \in A} f P_{0,a}^{\text{buy}}(\omega) x_{0,a}^{+}(\omega) = W_0 \qquad \omega \in \Omega \tag{3}$$

$$\sum_{a\in A\backslash\{S\}} \frac{1}{2}\delta^{a}_{t-1}(\omega)F^{a}x^{-}_{t,a}(\omega) + \sum_{a\in A} gP^{\text{sell}}_{t,a}(\omega)x^{-}_{t,a}(\omega) = \sum_{a\in A} fP^{\text{buy}}_{t,a}(\omega)x^{+}_{t,a}(\omega) \qquad \omega\in\Omega \quad t\in T^{d}\backslash\{0\}\,. \tag{4}$$

In (4) the left-hand side represents the cash freed up to be reinvested at time $t \in T^{d}\backslash\{0\}$ and consists of two distinct components. The first term represents the semi-annual coupons received on the coupon-bearing Treasury bonds held between time $t-1$ and t, the second term represents the cash obtained from selling part of the portfolio. This must equal the value of the new assets bought given by the right hand side of (4).

Table 1 Variables and parameters of the model

Time Sets	
$T^{\text{total}} = \left\{0, \frac{1}{12}, \ldots, T\right\}$	*set of all times* considered in the stochastic programme
$T^{d} = \{0, 1, \ldots, T-1\}$	*set of decision times*
$T^{i} = T^{\text{total}}\backslash T^{d}$	*set of intermediate times*
$T^{c} = \left\{\frac{1}{2}, \frac{3}{2}, \ldots, T-\frac{1}{2}\right\}$	*times when a coupon is paid out in-between decision times*
Instruments	
$S_t(\omega)$	*Dow Jones EuroStoxx 50 index level* at time t in scenario ω
$B^{T}_{t}(\omega)$	*EU Treasury bond with maturity T* at time t in scenario ω
$\delta^{B^T}_{t}(\omega)$	*coupon rate* of EU Treasury bond with maturity T at time t in scenario ω
F^{B^T}	*face value* of EU Treasury bond with maturity T
$Z_t(\omega)$	*EU zero-coupon Treasury bond price* at time t in scenario ω
Risk Management Barrier	
$y_{t,T}(\omega)$	*EU zero-coupon Treasury yield* with maturity T at time t in scenario ω
G	*annual guaranteed return*
$L^{N}_{t}(\omega)$	*nominal barrier* at time t in scenario ω
Portfolio Evolution	
A	*set of all assets*
$P^{buy}_{t,a}(\omega)\,/\,P^{sell}_{t,a}(\omega)$	*buy/sell price* of asset $a \in A$ at time t in scenario ω
f/g	*transaction costs* for buying / selling
$x_{t,a}(\omega)$	*quantity held* of asset $a \in A$ between time t and $t + 1/12$ in scenario ω
$x^{+}_{t,a}(\omega)\,/\,x^{-}_{t,a}(\omega)$	*quantity bought/sold* of asset $a \in A$ at time t in scenario ω
W_0	*initial portfolio wealth*
$W_t(\omega)$	*portfolio wealth before rebalancing* at time $t \in T$ in scenario ω
$w_t(\omega)$	*portfolio wealth after rebalancing* at time $t \in T^{c} \cup T^{d}\backslash\{T\}$ in scenario ω
$h_t(\omega) := \max(0, L_t(\omega) - W_t(\omega))$	*shortfall* at time t in scenario ω

- *short sale constraints*. In our model we assume no short selling of any stocks or bonds

$$x_{t,a}(\omega) \geq 0 \qquad a \in A \quad \omega \in \Omega \quad t \in T^{\text{total}} \tag{5}$$

$$x^{+}_{t,a}(\omega) \geq 0 \qquad \forall a \in A \quad \forall \omega \in \Omega \quad \forall t \in T^{\text{total}} \backslash \{T\} \tag{6}$$

$$x^{-}_{t,a}(\omega) \geq 0 \qquad \forall a \in A \quad \forall \omega \in \Omega \quad \forall t \in T^{\text{total}} \backslash \{0\}. \tag{7}$$

- *information constraints.* These constraints ensure that the portfolio allocation can not be changed during the period from one decision time to the next and hence that no decisions with perfect foresight can be made

$$x^{+}_{t,a}(\omega) = x^{-}_{t,a}(\omega) = 0 \qquad a \in A \quad \omega \in \Omega \quad t \in T^{i} \backslash T^{c}. \tag{8}$$

- *wealth constraint.* This constraint determines the portfolio wealth at each point in time

$$w_t(\omega) = \sum_{a \in A} P^{\text{buy}}_{t,a}(\omega) x_{t,a}(\omega) \qquad \omega \in \Omega \quad t \in T^{\text{total}} \backslash \{T\} \tag{9}$$

$$W_t(\omega) = \sum_{a \in A} P^{\text{sell}}_{t,a}(\omega) x_{t-\frac{1}{12},a}(\omega) \qquad \omega \in \Omega \quad t \in T^{\text{total}} \backslash \{0\} \tag{10}$$

$$w_T(\omega) = \sum_{a \in A} g P^{\text{sell}}_{T,a}(\omega) x_{T-\frac{1}{12},a}(\omega) + \sum_{a \in A \backslash \{S\}} \frac{1}{2} \delta^{a}_{T-1}(\omega) F^{a} x_{T-\frac{1}{12},a}(\omega) \quad \omega \in \Omega. \tag{11}$$

- *accounting balance constraints.* These constraints give the quantity invested in each asset at each time and for each scenario

$$x_{0,a}(\omega) = x^{+}_{0,a}(\omega) \qquad a \in A \quad \omega \in \Omega \tag{12}$$

$$x_{t,a}(\omega) = x_{t-\frac{1}{12},a}(\omega) + x^{+}_{t,a}(\omega) - x^{-}_{t,a}(\omega) \qquad a \in A \quad \omega \in \Omega \quad t \in T^{\text{total}} \backslash \{0\}. \tag{13}$$

The total quantity invested in asset $a \in A$ between time $t$and $t + \frac{1}{12}$ is equal to the total quantity invested in asset $a \in A$ between time $t - \frac{1}{12}$ and t plus the quantity of asset $a \in A$ bought at time t minus the quantity of asset $a \in A$ sold at time t.

- *annual rolling constraint*. This constraint ensures that at each decision time all the coupon-bearing Treasury bond holdings are sold

$$x^{-}_{t,a}(\omega) = x_{t-\frac{1}{12},a}(\omega) \qquad a \in A \backslash \{S\} \qquad \omega \in \Omega \qquad t \in T^{d} \backslash \{0\}. \tag{14}$$

- *coupon re-investment constraints.* We assume that the coupon paid every six months will be re-invested in the same coupon-bearing Treasury bond

$$x^+_{t,a}(\omega) = \frac{\frac{1}{2}\delta^a_t(\omega)F^a x_{t-\frac{1}{12},a}(\omega)}{f P^{\mathrm{buy}}_{t,a}(\omega)} \qquad x^-_{t,a}(\omega) = 0$$

$$x^+_{t,S}(\omega) = x^-_{t,S}(\omega) = 0 \tag{15}$$

$$a \in A\backslash\{S\} \quad \omega \in \Omega \quad t \in T^c.$$

- *barrier constraints.* These constraints determine the shortfall of the portfolio at each time and scenario as defined in Table 1

$$h_t(\omega) + W_t(\omega) \geq L_t(\omega) \qquad \omega \in \Omega \quad t \in T^{\mathrm{total}} \tag{16}$$

$$h_t(\omega) \geq 0 \qquad \omega \in \Omega \quad t \in T^{\mathrm{total}}. \tag{17}$$

As the objective of the stochastic programme will put a penalty on any shortfall, optimizing will ensure that $h_t\,(\omega)$ will be zero if possible and as small as possible otherwise, i.e.

$$h_t\,(\omega) = \max\,(0, L_t\,(\omega) - W_t\,(\omega)) \qquad \forall\omega \in \Omega \quad \forall t \in T^{\mathrm{total}} \tag{18}$$

which is exactly how we defined $h_t\,(\omega)$ in (2).

To obtain the maximum shortfall for each scenario, we need to add one of the following two sets of constraints:

$$H\,(\omega) \geq h_t\,(\omega) \qquad \forall\omega \in \Omega \quad \forall t \in T^{\mathrm{d}} \cup \{T\} \tag{19}$$

$$H\,(\omega) \geq h_t\,(\omega) \qquad \forall\omega \in \Omega \quad \forall t \in T^{\mathrm{total}} \tag{20}$$

Constraint (19) needs to be added if the max shortfall is to be taken into account on a yearly basis and constraint (20) if max shortfall is on a monthly basis.

2.3 Objective Functions: Expected Average Shortfall and Expected Maximum Shortfall

Starting with an initial wealth W_0 and an annual *nominal guarantee* of G, the liability at the planning horizon at time T is given by

$$W_0\,(1+G)^T\,. \tag{21}$$

To price the liability at time $t < T$ consider a zero-coupon Treasury bond, which pays 1 at time T, i.e. $Z_T(\omega) = 1$, for all scenarios $\omega \in \Omega$. The *zero-coupon Treasury bond price* at time t in scenario ω assuming continuous compounding is given by

$$Z_t\,(\omega) = e^{-y_{t,T}(\omega)(T-t)}, \tag{22}$$

where $y_{t,T}(\omega)$ is the *zero-coupon Treasury yield* with maturity T at time t in scenario ω.

This gives a formula for the value of the *nominal or fixed guarantee barrier* at time t in scenario ω as

$$L_t^N(\omega) := W_0(1+G)^T Z_t(\omega) = W_0(1+G)^T e^{-y_{t,T}(\omega)(T-t)}. \tag{23}$$

In a minimum guaranteed return fund the objective of the fund manager is twofold; firstly to manage the investment strategies of the fund and secondly to take into account the guarantees given to all investors. Investment strategies must ensure that the guarantee for all participants of the fund is met with a high probability.

In practice the guarantor (the parent bank of the fund manager) will ensure the investor guarantee is met by forcing the purchase of the zero coupon bond of (22) when the fund is sufficiently near the barrier defined by (23). Since all upside potential to investors is thus foregone, the aim of the fund manager is to fall below the barrier with acceptably small if not zero probability.

Ideally we would add a constraint limiting the probability of falling below the barrier in a VaR-type minimum guarantee constraint, i.e.

$$P\left(\max_{t\in T^{\text{total}}} h_t(\boldsymbol{\omega}) > 0\right) \le \alpha \tag{24}$$

for α small. However, such scenario-based probabilistic constraints are extremely difficult to implement, as they may without further assumptions convert the convex large-scale optimization problem into a non-convex one. We therefore use the following two convex approximations in which we trade off the risk of falling below the barrier against the return in the form of the expected sum of wealth.

Firstly, we look at the *expected average shortfall* (EAS) model in which the objective function is given by:

$$\begin{aligned}
&\max_{\substack{\left\{x_{t,a}(\omega),x^+_{t,a}(\omega),x^-_{t,a}(\omega):\right. \\ \left.a\in A,\omega\in\Omega,t\in T^d\cup\{T\}\right\}}} \left\{\sum_{\omega\in\Omega}\sum_{t\in T^d\cup\{T\}} p(\omega)\left((1-\beta)\,W_t(\omega) - \beta\frac{h_t(\omega)}{|T^d\cup\{T\}|}\right)\right\} \\
&= \max_{\substack{\left\{x_{t,a}(\omega),x^+_{t,a}(\omega),x^-_{t,a}(\omega):\right. \\ \left.a\in A,\omega\in\Omega,t\in T^d\cup\{T\}\right\}}} \left\{(1-\beta)\left(\sum_{\omega\in\Omega} p(\omega)\sum_{t\in T^d\cup\{T\}} W_t(\omega)\right)\right. \\
&\qquad\qquad \left. -\,\beta\left(\sum_{\omega\in\Omega} p(\omega)\sum_{t\in T^d\cup\{T\}}\frac{h_t(\omega)}{|T^d\cup\{T\}|}\right)\right\}.
\end{aligned} \tag{25}$$

In this case we maximize the expected sum of wealth over time while penalizing each time the wealth falls below the barrier. For each scenario $\omega \in \Omega$ we can calculate the average shortfall over time and then take expectations over all scenarios.

In this case only shortfalls at decision times are taken into account and any serious loss in portfolio wealth in-between decision times is ignored. However from the fund manager's and guarantor's perspective the position of the portfolio wealth relative to the fund's barrier is significant on a continuous basis and serious or repeated drops below this barrier might force the purchase of expensive insurance. To capture this feature specific to minimum guaranteed return funds, we also consider an objective function in which the shortfall of the portfolio is considered on a monthly basis.

For the *expected average shortfall with monthly checking* (EAS MC) model the objective function is given by

$$\max_{\substack{\left\{x_{t,a}(\omega),x^{+}_{t,a}(\omega),x^{-}_{t,a}(\omega):\right.\\ \left.a\in A,\omega\in\Omega,t\in T^{d}\cup\{T\}\right\}}} \left\{(1-\beta)\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^{d}\cup\{T\}} W_t(\omega)\right) - \beta\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^{\text{total}}} \frac{h_t(\omega)}{\left|T^{\text{total}}\right|}\right)\right\}. \tag{26}$$

Note that although we still only rebalance once a year shortfall is now being measured on a monthly basis in the objective and hence the annual decisions must also take into account the possible effects they will have on the monthly shortfall.

The value of $0 \leq \beta \leq 1$ can be chosen freely and sets the level of risk aversion. The higher the value of β, the higher the importance given to shortfall and the less to the expected sum of wealth, and hence the more risk-averse the optimal portfolio allocation will be. The two extreme cases are represented by $\beta = 0$, corresponding to the 'unconstrained' situation, which is indifferent to the probability of falling below the barrier, and $\beta = 1$, corresponding to the situation in which the shortfall is penalized and the expected sum of wealth ignored.

In general short horizon funds are likely to attract more risk-averse participants than long horizon funds, whose participants can afford to tolerate more risk in the short run. This natural division between short and long-horizon funds is automatically incorporated in the problem set up, as the barrier will initially be lower for long-term funds than for short-term funds as exhibited in Figure 3. However the importance of closeness to the barrier can be adjusted by the choice of β in the objective.

The second model we consider is the *expected maximum shortfall* (EMS) model given by:

$$\max_{\substack{\left\{x_{t,a}(\omega),x^{+}_{t,a}(\omega),x^{-}_{t,a}(\omega):\right.\\ \left.a\in A,\omega\in\Omega,t\in T^{d}\cup\{T\}\right\}}} \left\{(1-\beta)\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^{d}\cup\{T\}} W_t(\omega)\right) - \beta\left(\sum_{\omega\in\Omega} p(\omega) H(\omega)\right)\right\} \tag{27}$$

using the constraints (19) to define $H(\omega)$.

For the *expected maximum shortfall with monthly checking* (EMS MC) model the objective function remains the same but $H(\omega)$ is now defined by (20).

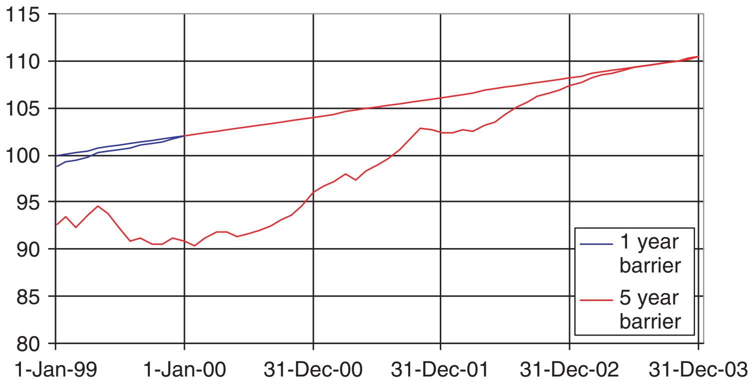

Fig. 3 Barrier for one-year and five-year 2% guaranteed return fund

In both variants of this model we penalize the expected maximum shortfall, which ensures that $H(\omega)$ is as small as possible for each scenario $\omega \in \Omega$. Combining this with constraints (19)/(20) it follows that $H(\omega)$ is exactly equal to the maximum shortfall.

The constraints given in Section 2.2 apply to both the expected average shortfall and expected maximum shortfall models.

The EAS model incurs a penalty every time portfolio wealth falls below the barrier, but it does not differentiate between a substantial shortfall at one point in time and a series of small shortfalls over time. The EMS model on the other hand, focusses on limiting the maximum shortfall and therefore does not penalize portfolio wealth falling just slightly below the barrier several times. So one model limits the *number of times* portfolio wealth falls below the barrier while the other limits *any shortfall substantially*.

3 Bond Pricing

In this section we present a three-factor term structure model which we will use to price both our bond portfolio and the fund's liability. Many interest-rate models, like the classic one-factor Vasicek (1977) and Cox, Ingersoll, and Ross (1985) class of models and even more recent multi-factor models like Anderson and Lund (1997), concentrate on modeling just the short-term rate.

However for the minimum guaranteed return funds we have to deal with a long-term liability and bonds of varying maturities. We therefore must capture the dynamics of the whole term structure. This has been achieved by using the economic factor model described below in Section 3.1. In Section 3.2 we describe the pricing of coupon-bearing bonds and Section 3.3 investigates the consequences of rolling the bonds on an annual basis.

3.1 Yield Curve Model

To capture the dynamics of the whole term structure, we will use a Gaussian *economic factor model* (EFM) (see Campbell (2000) and also Nelson and Siegel (1987)) whose evolution under the risk-neutral measure Q is determined by the stochastic differential equations

$$\mathbf{dX}_t = (\mu_X - \lambda_X X_t)\,dt + \sigma_X \mathbf{dW}_t^X \tag{28}$$

$$\mathbf{dY}_t = (\mu_Y - \lambda_Y Y_t)\,dt + \sigma_Y \mathbf{dW}_t^Y \tag{29}$$

$$\mathbf{dR}_t = k\,(X_t + Y_t - R_t)\,dt + \sigma_R \mathbf{dW}_t^R \tag{30}$$

where the $\boldsymbol{dW}$ terms are correlated. The three unobservable Gaussian factors $\mathbf{R}$, $\mathbf{X}$ and $\mathbf{Y}$ represent respectively a *short rate*, a *long rate* and (minus) the *slope* between an *instantaneous short rate* and the long rate. Solving these equations the following formula for the yield at time t with time to maturity equal to $T-t$ is obtained (for a derivation, see Medova *et al.*, 2005)

$$y_{t,T} = \frac{A\,(t,T)\,R_t + B\,(t,T)\,X_t + C\,(t,T)\,Y_t + D\,(t,T)}{T}, \tag{31}$$

where

$$A\,(t,T) := \frac{1}{k}\left(1 - e^{-k(T-t)}\right) \tag{32}$$

$$B\,(t,T) := \frac{k}{k-\lambda_X}\left\{\frac{1}{\lambda_X}\left(1 - e^{-\lambda_X(T-t)}\right) - \frac{1}{k}\left(1 - e^{-k(T-t)}\right)\right\} \tag{33}$$

$$C\,(t,T) := \frac{k}{k-\lambda_Y}\left\{\frac{1}{\lambda_Y}\left(1 - e^{-\lambda_Y(T-t)}\right) - \frac{1}{k}\left(1 - e^{-k(T-t)}\right)\right\} \tag{34}$$

$$\begin{aligned}
D\,(t,T) :=& \left(T - t - \frac{1}{k}\left(1 - e^{-k(T-t)}\right)\right)\left(\frac{\mu_X}{\lambda_X} + \frac{\mu_Y}{\lambda_Y}\right) - \frac{\mu_X}{\lambda_X}B\,(t,T) - \frac{\mu_Y}{\lambda_Y}C\,(t,T)\\
&-\frac{1}{2}\sum_{i=1}^{3}\left\{\frac{m_{X_i}}{2\lambda_X}\left(1 - e^{-2\lambda_X(T-t)}\right) + \frac{m_{Y_i}}{2\lambda_Y}\left(1 - e^{-2\lambda_Y(T-t)}\right)\right.\\
&+\frac{n_i^2}{2k}\left(1 - e^{-2k(T-t)}\right) + p_i^2\,(T-t) + \frac{2m_{X_i}m_{Y_i}}{\lambda_X+\lambda_Y}\left(1 - e^{-(\lambda_X+\lambda_Y)(T-t)}\right)\\
&+\frac{2m_{X_i}n_i}{\lambda_X+k}\left(1 - e^{-(\lambda_X+k)(T-t)}\right) + \frac{2m_{X_i}p_i}{\lambda_X}\left(1 - e^{-\lambda_X(T-t)}\right)\\
&+\frac{2m_{Y_i}n_i}{\lambda_Y+k}\left(1 - e^{-(\lambda_Y+k)(T-t)}\right) + \frac{2m_{Y_i}p_i}{\lambda_Y}\left(1 - e^{-\lambda_Y(T-t)}\right)\\
&\left.+\ \frac{2n_i p_i}{k}\left(1 - e^{-k(T-t)}\right)\right\}
\end{aligned} \tag{35}$$

and

$$
\begin{aligned}
m_{X_i} &:= -\frac{k\sigma_{X_i}}{\lambda_X\,(k-\lambda_X)} \\
m_{Y_i} &:= -\frac{k\sigma_{Y_i}}{\lambda_Y\,(k-\lambda_Y)} \\
n_i &:= \frac{\sigma_{X_i}}{k-\lambda_X} + \frac{\sigma_{Y_i}}{k-\lambda_Y} - \frac{\sigma_{R_i}}{k} \\
p_i &:= -\left(m_{X_i} + m_{Y_i} + n_i\right).
\end{aligned}
\tag{36}
$$

Bond pricing must be effected under the *risk-neutral* measure Q. However, for the model to be used for forward simulation the set of stochastic differential equations must be adjusted to capture the model dynamics under the real-world or *market* measure P. We therefore have to model the market prices of risk which take us from the risk-neutral measure Q to the real-world measure P.

Under the market measure P we adjust the drift term by adding the *risk premium* given by the *market price of risk* γ in terms of the quantity of risk. The effect of this is a change in the long-term mean, e.g. for the factor $\mathbf{X}$ the long-term mean now equals $\frac{\mu_X+\gamma_X\sigma_X}{\lambda_X}$. It is generally assumed in a Gaussian world that the quantity of risk is given by the *volatility* of each factor.

This gives us the following set of processes under the market measure

$$d\mathbf{X}_t = (\mu_X - \lambda_X X_t + \gamma_X\sigma_X)dt + \sigma_X d\mathbf{W}_t^X \tag{37}$$

$$d\mathbf{Y}_t = (\mu_Y - \lambda_Y Y_t + \gamma_Y\sigma_Y)dt + \sigma_Y d\mathbf{W}_t^Y \tag{38}$$

$$d\mathbf{R}_t = \{k(X_t + Y_t - R_t) + \gamma_R\sigma_R\}dt + \sigma_R d\mathbf{W}_t^R, \tag{39}$$

where all three factors contain a market price of risk γ in volatility units.

The yields derived in the economic factor model are continuously compounded while most yield data are annually compounded. So for appropriate comparison when estimating the parameters of the model we will have to convert the annually compounded yields into continuously compounded yields using the transformation

$$y^{(\text{continuous})} = \ln(1 + y^{(\text{annual})}). \tag{40}$$

In the limit as T tends to infinity it can be shown that expression (31) derived for the yield does not tend to the 'long rate' factor X, but to a constant. This suggests that the three factors introduced in this term structure model may really be unobservable. To handle the unobservable state variables we formulate the model in state space form, a detailed description of which can be found in Harvey (1993) and use the Kalman filter to estimate the parameters (see e.g. Dempster *et al.*, 1999).

3.2 Pricing Coupon-Bearing Bonds

As sufficient historical data on Euro coupon-bearing Treasury bonds is difficult to obtain we use the zero-coupon yield curve to construct the relevant bonds. Coupons on newly-issued bonds are generally closely related to the corresponding spot rate at the time, so the current zero-coupon yield with maturity T is used as a proxy for the coupon rate of a coupon-bearing Treasury bond with maturity T. For example, on scenario ω the coupon rate $\delta_2^{B^{10}}(\omega)$ on a newly issued 10-year Treasury bond at time $t = 2$ will be set equal to the projected 10-year spot rate $y_{2,10}(\omega)$ at time $t = 2$.

Generally

$$\delta_t^{B^{(T)}}(\omega) = y_{t,T}(\omega) \qquad \forall t \in T^d \qquad \forall \omega \in \Omega \tag{41}$$

$$\delta_t^{B^{(T)}}(\omega) = \delta_{\lfloor t \rfloor}^{(T)}(\omega) \qquad \forall t \in T^i \qquad \forall \omega \in \Omega, \tag{42}$$

where $\lfloor . \rfloor$ denotes integral part. This ensures that as the yield curve falls, coupons on newly-issued bonds will go down correspondingly and each coupon cash flow will be discounted at the appropriate zero-coupon yield.

The bonds are assumed to pay coupons semi-annually. Since we roll the bonds on an annual basis, a coupon will be received after six months and again after a year just before the bond is sold. This forces us to distinguish between the price at which the bond is sold at rebalancing times and the price at which the new bond is purchased.

Let $P_{t,B^{(T)}}^{(\text{sell})}$ denote the selling price of the bond $B^{(T)}$ at time t, assuming two coupons have now been paid out and the time to maturity is equal to $T-1$, and let $P_{t,B^{(T)}}^{(\text{buy})}$ denote the price of a newly issued coupon-bearing Treasury bond with a maturity equal to T.

The 'buy' bond price at time t is given by

$$\begin{aligned} B_t^T(\omega) &= F^{B^T} e^{-(T+\lfloor t \rfloor - t) y_{t,T+\lfloor t \rfloor - t}(\omega)} \\ &\quad + \sum_{s=\frac{\lfloor 2t \rfloor}{2}+\frac{1}{2},\frac{\lfloor 2t \rfloor}{2}+1,\ldots,\lfloor t \rfloor + T} \frac{\delta_t^{B^T}(\omega)}{2} F^{B^T} e^{-(s-t) y_{t,(s-t)}(\omega)} \\ &\omega \in \Omega \quad t \in T^{\text{total}}, \end{aligned} \tag{43}$$

where the principal of the bond is discounted in the first term and the stream of coupon payments in the second.

At rebalancing times t the sell price of the bond is given by

$$\begin{aligned} B_t^T(\omega) &= F^{B^T} e^{-(T-1) y_{t,T-1}(\omega)} + \sum_{s=\frac{1}{2},1,\ldots,T-1} \frac{\delta_{t-1}^{B^T}(\omega)}{2} F^{B^T} e^{-(s-t) y_{t,(s-t)}(\omega)} \\ &\omega \in \Omega \quad t \in \left\{T^d \setminus \{0\}\right\} \cup \{T\} \end{aligned} \tag{44}$$

with coupon rate $\delta_{t-1}^{B^T}(\omega)$. The coupon rate is then reset for the newly-issued Treasury bond of the same maturity. We assume that the coupons paid at six months are re-invested in the off-the-run bonds. This gives the following adjustment to the amount held in bond B^T at time t

$$x_{t,B^T}(\omega) = x_{t-\frac{1}{12},B^T}(\omega) + \frac{\frac{1}{2}\delta_t^{B^T}(\omega)F^{B^T}x_{t-\frac{1}{12},B^T}(\omega)}{fP_{t,B^T}^{\text{buy}}(\omega)} \quad t \in T^c \quad \omega \in \Omega. \tag{45}$$

4 Historical Backtests

We will look at an *historical backtest* in which statistical models are fitted to data up to a trading time t and scenario trees are generated to some chosen horizon $t+T$. The optimal root node decisions are then implemented at time t and compared to the historical returns at time t + 1. Afterwards the whole procedure is rolled forward for T trading times.

Our backtest will involve a *telescoping horizon* as depicted in Figure 4.

At each decision time t the parameters of the stochastic processes driving the stock return and the three factors of the term structure model are re-calibrated using historical data up to and including time t and the initial values of the simulated scenarios are given by the actual historical values of the variables at these times. Re-calibrating the simulator parameters at each successive initial decision time t captures information in the history of the variables up to that point.

Although the optimal second and later-stage decisions of a given problem may be of "what-if" interest, manager and decision maker focus is on the implementable first-stage decisions which are hedged against the simulated future uncertainties. The reasons for implementing stochastic optimization programmes in this way are twofold. Firstly, after one year has passed the actual values of the variables realized may not coincide with any of the values of the variables in the simulated scenarios. In this case the optimal investment policy would be undefined, as the model only has

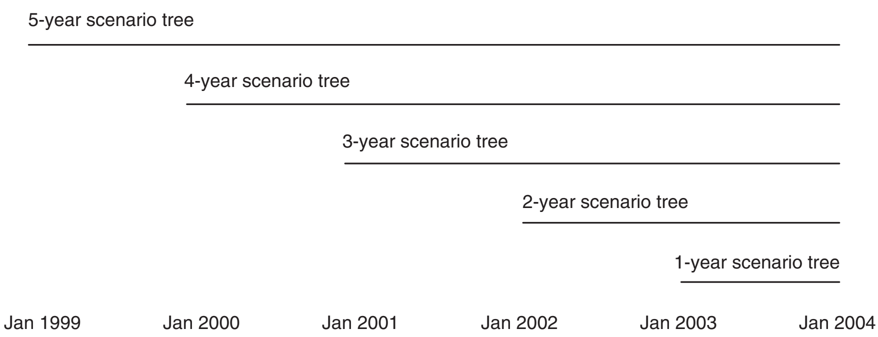

Fig. 4 Telescoping horizon backtest schema

optimal decisions defined for the nodes on the simulated scenarios. Secondly, as one more year has passed new information has become available to re-calibrate the simulator's parameters. Relying on the original optimal investment strategies will ignore this information. For more on backtesting procedures for stochastic optimization models see Dempster *et al.* (2003).

For our backtests we will use three different tree structures with approximately the same number of scenarios, but with an increasing initial branching factor. We first solve the five-year problem using a 6.6.6.6.6 tree, which gives 7776 scenarios. Then we use 32.4.4.4.4 = 8192 scenarios and finally the extreme case of 512.2.2.2.2 = 8192 scenarios.

For the subsequent stages of the telescoping horizon backtest we adjust the branching factors in such a way that the total number of scenarios stays as close as possible to the original number of scenarios and the same ratio is maintained. This gives us the tree structures set out in Table 2.

Historical backtests show how specific models would have performed had they been implemented in practice. The reader is referred to Rietbergen (2005) for the calibrated parameter values employed in these tests. Figures 5 to 10 show how the various optimal portfolios' wealth would have evolved historically relative to the barrier. It is also important to determine how the models performed in-sample on the generated scenario trees and whether or not they had realistic forecasts with regard to future historical returns. To this end one-year-ahead in-sample expectations of

Table 2 Tree structures for different horizon backtests

Jan 1999	6.6.6.6.6 = 7776	32.4.4.4.4 = 8192	512.2.2.2.2 = 8192
Jan 2000	9.9.9.9 = 6561	48.6.6.6 = 10368	512.2.2.2 = 4096
Jan 2001	20.20.20 = 8000	80.10.10 = 8000	768.3.3 = 6912
Jan 2002	88.88 = 7744	256.32 = 8192	1024.8 = 8192
Jan 2003	7776	8192	8192

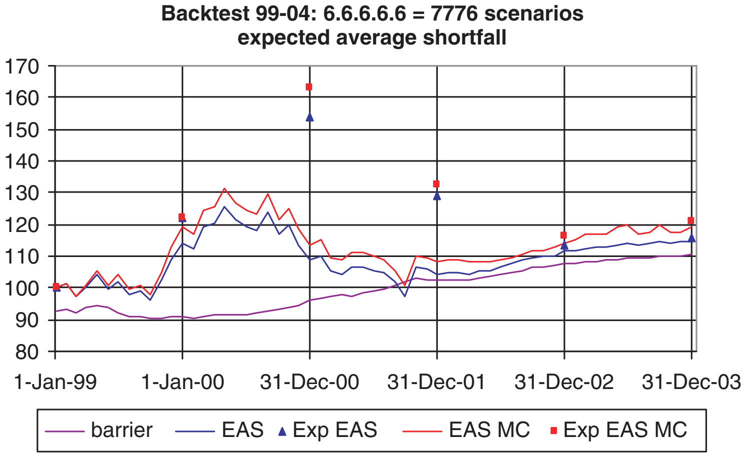

Fig. 5 Backtest 1999-2004 using expected average shortfall for the 6.6.6.6.6 tree

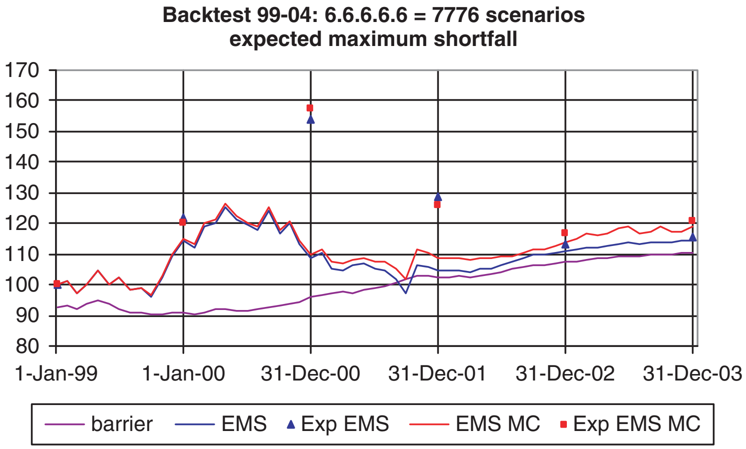

Fig. 6 Backtest 1999-2004 using expected maximum shortfall for the 6.6.6.6.6 tree

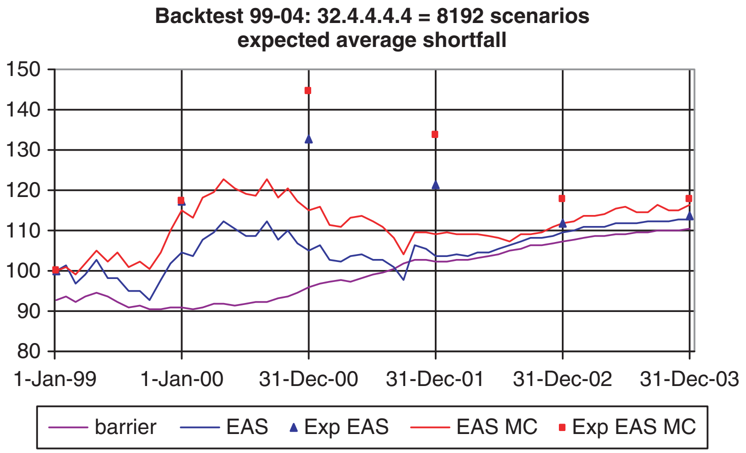

Fig. 7 Backtest 1999-2004 using expected average shortfall for the 32.4.4.4.4 tree

portfolio wealth are shown as points in the backtest performance graphs. Implementing the first-stage decisions *in-sample* the portfolio's wealth is calculated one year later for each scenario in the simulated tree after which an expectation is taken over the scenarios.

From these graphs a first observation is that the risk management monitoring incorporated into the model appears to work well. In all cases the only time portfolio wealth dips below the barrier, if at all, is on September 11, 2001. The initial in-sample wealth overestimation of the models is likely to be due mainly to the short time series available for initial parameter estimation which led to hugely inflated stock return expectations during the equity bubble. However as time progresses and

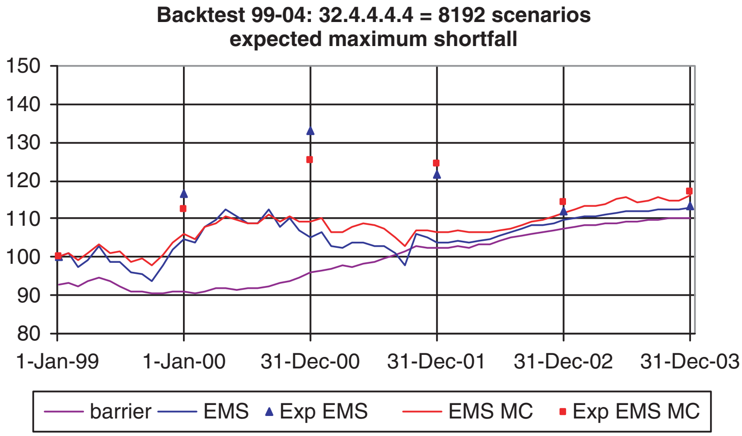

Fig. 8 Backtest 1999-2004 using expected maximum shortfall for the 32.4.4.4.4 tree

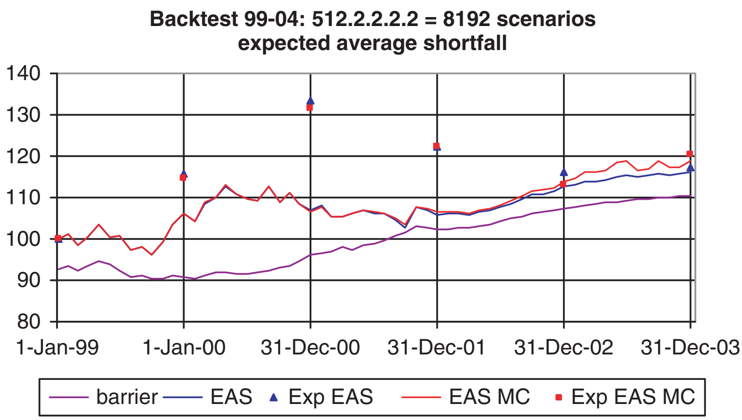

Fig. 9 Backtest 1999-2004 using expected average shortfall for the 512.2.2.2.2 tree

more data points to re-calibrate the model are obtained, the models' expectations and real-life realizations very closely approximate each other.

For reference we have included the performance of the EuroStoxx 50 in Figure 11 to indicate the performance of the stock market over the backtesting period. Even though this was a difficult period for the optimal portfolios to generate high historical returns, it provides an excellent demonstration that the risk management incorporated into the models operates effectively. It is in periods of economic downturn that one wants the portfolio returns to survive.

Tables 3 and 4 give the optimal portfolio allocations for the 32.4.4.4.4 tree using the two maximum shortfall objective functions. In both cases we can observe a tendency for the portfolio to move to the safer, shorter-term assets as time progresses. This is naturally built into the model as depicted in Figure 3.

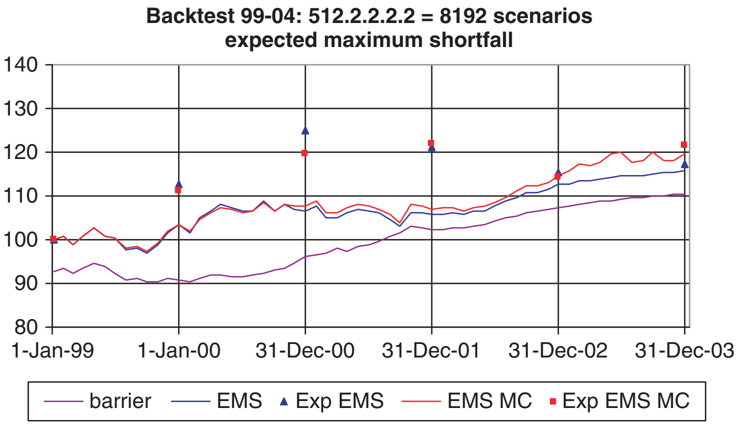

Fig. 10 Backtest 1999-2004 using expected maximum shortfall for the 512.2.2.2.2 tree

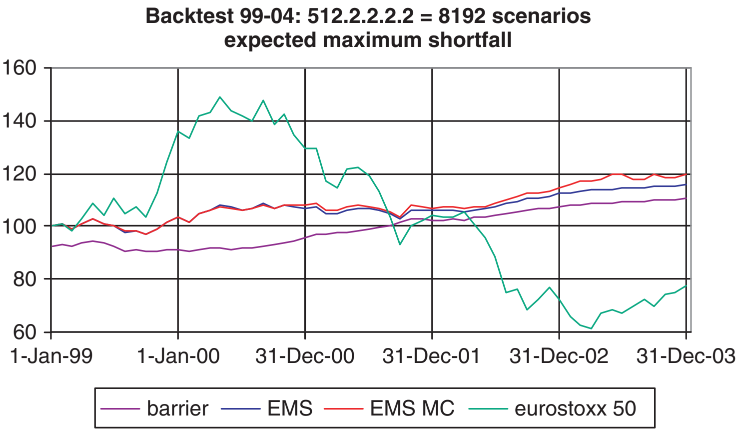

Fig. 11 Comparison of the fund's portfolio performance to the eurostoxx 50

Table 3 Portfolio allocation expected maximum shortfall using the 32.4.4.4.4 tree

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	0	**0.23**	**0.45**	**0.32**
Jan 00	0	0	0	0	0	0	**0.37**	**0.63**
Jan 01	**0.04**	0	0	0	0	**0.39**	**0.53**	**0.40**
Jan 02	**0.08**	**0.16**	**0.74**	0	0	0	0	**0.01**
Jan 03	**0.92**	0	0	0	0	**0.07**	0	**0.01**

Table 4 Portfolio allocation expected Maximum shortfall with monthly checking using the 32.4.4.4.4 tree

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	**0.49**	**0.27**	0	**0.24**
Jan 00	0	0	0	0	**0.25**	**0.38**	0	**0.36**
Jan 01	0	0	0	0	**0.49**	**0.15**	0	**0.36**
Jan 02	0	0	0	**0.47**	**0.44**	0	0	**0.10**
Jan 03	0	0	**0.78**	**0.22**	0	0	0	**0.01**

For the decisions made in January 2002/2003, the portfolio wealth is significantly closer to the barrier for the EMS model than it is for the EMS MC model. This increased risk for the fund is taken into account by the EMS model and results in an investment in safer short-term bonds and a negligible equity component. Whereas the EMS model stays in the one to three year range the EMS MC model invests mainly in bonds with a maturity in the range of three to five years and for both models the portfolio wealth manages to stay above the barrier.

From Figures 5 to 10 it can be observed that in all cases the method with monthly checking outperforms the equivalent method with just annual shortfall checks. Similarly as the initial branching factor is increased, the models' out-of-sample performance is generally improved. For the $512.2.2.2.2 = 8192$ scenario tree, all four objective functions give optimal portfolio allocations which keep the portfolio wealth above the barrier at all times, but the models with the monthly checking still outperform the others. The more important difference however seems to lie in the deviation of the expected in-sample portfolio's wealth from the actual historical realization of the portfolio value. Table 5 displays this annual deviation averaged over the five rebalances and shows a clear reduction in this deviation for three of the four models as the initial branching factor is increased. Again the model that uses the expected maximum shortfall with monthly checking as its objective function outperforms the rest.

Overall the historical backtests have shown that the described stochastic optimization framework carefully considers the risks created by the guarantee. The EMS MC model produces well-diversified portfolios that do not change drastically from one year to the next and results in optimal portfolios which even through a period of economic downturn and uncertainty remained above the barrier.

Table 5 Average annual deviation

	EAS	EAS MC	EMS	EMS MC
6.6.6.6.6	9.87%	13.21%	9.86%	10.77%
32.4.4.4.4	10.06%	9.41%	9.84%	7.78%
512.2.2.2.2	10.22%	8.78%	7.78%	6.86%

5 Robustness of Backtest Results

Empirical equity returns are now well known not to be normally distributed but rather to exhibit complex behaviour including fat tails. To investigate how the EMS MC model performs with more realistic asset return distributions we report in this section experiments using a geometric Brownian motion with Poisson jumps to model equity returns. The stock price process $\mathbf{S}$ is now assumed to follow

$$\frac{d\mathbf{S}_t}{\mathbf{S}_t} = \tilde{\mu}_S dt + \tilde{\sigma}_S d\tilde{\mathbf{W}}_t^S + \mathbf{J}_t d\mathbf{N}_t, \tag{46}$$

where $\mathbf{N}$ is an independent Poisson process with intensity λ and the jump saltus $\mathbf{J}$ at Poisson epochs is a normal random variable.

As the EMS MC model and the 512.2.2.2.2 tree provided the best results with Gaussian returns the backtest is repeated for this model and treesize. Figure 12 gives the historical backtest results and Tables 5 and 6 represent the allocations for the 512.2.2.2.2 tests with the EMS MC model for the original GBM process and the GBM with Poisson jumps process respectively. The main difference in the two tables is that the investment in equity is substantially lower initially when the equity index volatility is high (going down to 0.1% when the volatility is 28% in 2001), but then increases as the volatility comes down to 23% in 2003. This is born out by Figure 12 which shows much more realistic in-sample one-year-ahead portfolio wealth predictions (*cf.* Figure 10) and a 140 basis point increase in terminal historical fund return over the Gaussian model. These phenomena are the result of the calibration of the normal jump saltus distributions to have negative means and hence more downward than upwards jumps resulting in downwardly skewed equity index return distributions, but with the same compensated drift as in the GBM case.

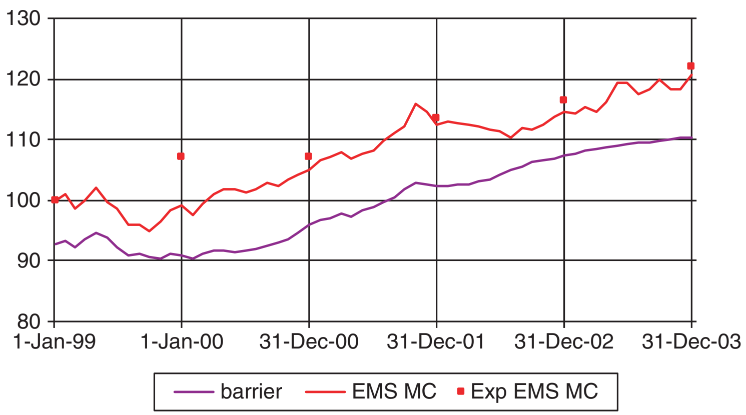

Fig. 12 Expected maximum shortfall with monthly checking using 512.2.2.2.2 tree for the GBM with jumps equity index process

Table 6 Portfolio allocation expected maximum shortfall with monthly checking using the 512.2.2.2.2 tree

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	**0.69**	**0.13**	0	**0.18**
Jan 00	0	0	0	0	**0.63**	0	0	**0.37**
Jan 01	0	0	0	0	**0.37**	**0.44**	0	**0.19**
Jan 02	0	0	0	0	**0.90**	0	0	**0.10**
Jan 03	0	0	**0.05**	0	**0.94**	0	0	**0.01**

Table 7 Portfolio allocation expected maximum shortfall with monthly checking using the 512.2.2.2.2 tree for the GBM with poisson jumps equity index process

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	**0.12**	**0.77**	0	**0.11**
Jan 00	0	0	0	0	0	**0.86**	0	**0.14**
Jan 01	0	0	0	0	**0.43**	**0.56**	0	**0.01**
Jan 02	0	0	0	0	**0.70**	**0.11**	0	**0.19**
Jan 03	0	0	0	0	**0.04**	**0.81**	0	**0.15**

As a consequence the optimal portfolios are more sensitive to equity variation and take benefit from its lower predicted value in the last year.

Although much more complex equity return processes are possible, these results show that the historical backtest performance of the EMS MC model is only improved in the presence of downwardly skewed asset equity return distributions possessing fat tails due to jumps.

6 Conclusions

This chapter has focussed on the design of funds to support investment products which give a minimum guaranteed return. We have concentrated here on the design of the liability side of the fund, paying particular attention to the pricing of bonds using a three-factor term structure model with reliable results for long-term as well as the short-term yields. Several objective functions for the stochastic optimization of portfolios have been constructed using expected average shortfall and expected maximum shortfall risk measures to combine risk management with strategic asset allocation. We also introduced the concept of monthly shortfall checking which improved the historical backtesting results considerably. In addition to the standard GBM model for equity returns we reported experiments using a GBM model with Poisson jumps to create downwardly skewed fat tailed equity index return distributions. The EMS MC model responded well with more realistic expected portfolio wealth predictions and the historical fund portfolio wealth staying significantly above the barrier at all times.

The models of this paper have been extended in practice to open ended funds which allow for contributions throughout the lifetime of the corresponding investment products. In total funds of the order of 10 billion euros have been managed with these extended models. In future research we hope to examine open multi-link pension funds constructed using several unit linked funds of varying risk aversion in order to allow the application of individual risk management to each client's portfolio.

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