

## INTRODUCTION

The study of epistemic attitudes – in particular knowledge and belief – dates at least back to the Scholasticism of the Middle Ages. The formal study of the same attitudes was then largely initiated by von Wright’s seminal paper from the 1950’s (37). The formal systematic study of knowledge and belief saw the light of day by Hintikka’s book by the same name *Knowledge and Belief: An Introduction to the Logic of the Two Notions* from 1962 (16). Hardly a publication in epistemic logic has surfaced since without reference to this ground-breaking investigation. More recent monographs dedicated to epistemic and/or doxastic logic<sup>1</sup> include notably Lenzen (22), Schlesinger (25), Boh (3), Knuuttila (19), Meyer and van der Hoek (35), Fagin et al. (6), Sowa (30), Hendricks (12), (14), and Halpern (11).

The contemporary logics of knowledge and belief are advanced and sophisticated. Epistemic and doxastics logics for single agents have been catalogued; logics for multi-agent systems have been catalogued; epistemic modalities have been combined with temporal and alethic modalities, etc. These advances make the way for multiple epistemic operators, multiple doxastic operators, common knowledge operators, alethic and temporal operators, mono-modal systems, multi-modal systems, dynamic epistemic/doxastic systems, belief revision features and agents equipped with learning mechanisms. This is not an exhaustive list. There is a vast fan of important applications and models utilizing these powerful logics of knowledge and belief. Examples range from robots on assembly lines, social and coalitional interactions, card games, ‘live’ situations in economics, miscellaneous linguistic practices and so on.

It is not the purpose of this introduction to review epistemic logic from its date of birth to this day and age in detail.<sup>2</sup> Certain distinctive developmental features stand out as particularly pertinent to both the research progression and direction as well as the general epistemological and applicational relevance of epistemic logic. These may be subsumed under ‘agent and system’, ‘active agenthood’, ‘multiple active agents’, ‘multi-modalities’ and constitute the features with respect to which this introduction and the contributions in this volume are organized.

## 1. AGENT AND SYSTEM

The formal systematic surveys of epistemic and doxastic logics were early on largely influenced by the advances in (alethic) modal logic. Standard systems of modal logic were furnished with epistemic interpretations, and some fundamental results of epistemic logic could then be extracted.

Syntactically, the language of propositional epistemic logic is obtained by augmenting the language of propositional logic with a unary epistemic

operator  $K_{\Xi}$  such that

$$K_{\Xi}A \text{ reads 'Agent } \Xi \text{ knows } A'$$

and similarly for belief

$$B_{\Xi}A \text{ reads 'Agent } \Xi \text{ believes } A'$$

for some arbitrary proposition  $A$ . These formalizations may be viewed as interpretations of  $\Box A$  in alethic logic reading 'It is necessary that  $A$ '. Interpreting modal logic epistemically is crudely a reading of modal formulae as epistemic statements expressing attitudes like knowledge, belief or conviction of certain agents towards certain propositions.

The semantics of modal logic is likewise given a novel interpretation. Hintikka came up with a semantic interpretation of epistemic and doxastic operators respectively which in terms of standard possible world semantics may be rendered accordingly (16):

$K_{\Xi}A$  : *in all possible worlds compatible with what  $\Xi$  knows it is the case that  $A$*

$B_{\Xi}A$  : *in all possible worlds compatible with what  $\Xi$  believes it is the case that  $A$*

The basic assumption is that any ascription of propositional attitudes like knowledge and belief, requires partitioning the set of possible worlds into two compartments: The compartment consisting of possible worlds compatible with the attitude in question and the compartment of worlds incompatible with it. Based on the partition the agent is capable of constructing different 'world-models' using the epistemic modal language. The agent is not necessarily required to know which one of the world-models constructed is the real world-model. Be that as it may, the agent does not consider all these world-models equally possible or accessible from his current point of view. Some world-models may be incommensurable with his current information state or other background assumptions. These incompatible world-models are excluded from the compatibility partition.<sup>3</sup>

The set of worlds considered accessible by an agent depends on the actual world, or the agent's actual state of information. It is possible to capture this dependency by introducing a relation of accessibility,  $R$ , on the set of compatible possible worlds. To express the idea that for agent  $\Xi$ , the world  $w'$  is compatible with his information state, or accessible from the possible world  $w$  which  $\Xi$  is currently in, it is required that  $R$  holds between  $w$  and  $w'$ . This relation is written  $Rww'$  and reads 'world  $w'$  is accessible from  $w$ '. The world  $w'$  is said to be an *epistemic* or *doxastic alternative* to world  $w$  for agent  $\Xi$  depending on whether knowledge or belief is the considered attitude. Given the above semantical interpretation, if a proposition  $A$  is true in all worlds which agent  $\Xi$  considers possible then  $\Xi$  knows  $A$  and similarly for belief.

A possible world semantics for a propositional epistemic logic with a single agent then consists of a *frame*  $\mathcal{F}$  which in turn is a pair  $(W, R)$  such that  $W$  is a non-empty set of possible worlds and  $R$  is a binary accessibility relation on  $W$ . A *model*  $\mathbb{M}$  for an epistemic system consists of a frame and a denotation function  $\varphi$  assigning sets of worlds to atomic propositional formulae. Propositions are taken to be sets of possible worlds; namely the set of possible worlds in which they are true. Let *atom* be the set of atomic propositional formulae, then  $\varphi : \text{atom} \rightarrow P(W)$  where  $P$  denotes the powerset operation. The model  $\mathbb{M} = \langle W, R, \varphi \rangle$  is called a Kripke-model and the resulting semantics Kripke-semantics (20): An atomic propositional formulae,  $\mathbf{a}$ , is said to be true in a world  $w$  (in  $\mathbb{M}$ ), written  $\mathbb{M}, w \models \mathbf{a}$ , iff  $w$  is in the set of possible worlds assigned to  $\mathbf{a}$ , i. e.  $\mathbb{M}, w \models \mathbf{a}$  iff  $w \in \varphi(\mathbf{a})$  for all  $\mathbf{a} \in \text{atom}$ . The formula  $K_{\Xi}A$  is true in a world  $w$ , i.e.  $\mathbb{M}, w \models K_{\Xi}A$ , iff  $\forall w' \in W : \text{if } Rww', \text{ then } \mathbb{M}, w' \models A$ . The semantics for the Boolean connectives follows the usual recursive recipe. A modal formula is said to be *valid* in a frame iff the formula is true for all possible assignments in all worlds admitted by the frame.

Similar semantics may be formulated for the belief operator. Since a belief is not necessarily true but rather probable, possible, or likely to be true belief may for instance be modelled by assigning a sufficiently high degree of probability to the proposition in question and determining the doxastic alternatives accordingly. The truth-conditions for the doxastic operator are defined in a way similar to that of the knowledge operator and the model may also be expanded to accommodate the two operators simultaneously.

A nice feature of possible world semantics is that many common epistemic axioms correspond to certain algebraic properties of the frame in the following sense: A modal axiom is valid in a frame if and only if the accessibility relation satisfies some algebraic condition. For an example, the axiom expressing the veridicality property that if a proposition is known by  $\Xi$ , then  $A$  is true,

$$(1) \quad K_{\Xi}A \rightarrow A,$$

is valid in all frames in which the accessibility relation is *reflexive* in the sense that

$$\forall w \in W : Rww.$$

Every possible world is accessible from itself. Similarly if the accessibility relation satisfies the condition that

$$\forall w, w', w'' \in W : Rww' \wedge Rww'' \rightarrow Rww''$$

then the axiom reflecting the idea that the agent knows that he knows  $A$  if he does,

$$(2) \quad K_{\Xi}A \rightarrow K_{\Xi}K_{\Xi}A,$$

is valid in all *transitive* frames. Other axioms of epistemic import require yet other rational properties to be met in order to be valid in all frames.

A nomenclature due to Lemmon (21) and later refined by Bull and Segerberg (4) is helpful while cataloguing the axioms typically considered interesting for epistemic logic (Table 1).

K	$K_{\Xi}(A \rightarrow A') \rightarrow (K_{\Xi}A \rightarrow K_{\Xi}A')$
D	$K_{\Xi}A \rightarrow \neg K_{\Xi}\neg A$
T	$K_{\Xi}A \rightarrow A$
4	$K_{\Xi}A \rightarrow K_{\Xi}K_{\Xi}A$
5	$\neg K_{\Xi}A \rightarrow K_{\Xi}\neg K_{\Xi}A$
.2	$\neg K_{\Xi}\neg K_{\Xi}A \rightarrow K_{\Xi}\neg K_{\Xi}\neg A$
.3	$K_{\Xi}(K_{\Xi}A \rightarrow K_{\Xi}A') \vee K_{\Xi}(K_{\Xi}A' \rightarrow K_{\Xi}A)$
.4	$A \rightarrow (\neg K_{\Xi}\neg K_{\Xi}A \rightarrow K_{\Xi}A)$

TABLE 1. Common epistemic axioms

- Axiom K, also called the *axiom of deductive cogency*: If the agent  $\Xi$  knows  $A \rightarrow A'$ , then if  $\Xi$  knows  $A$ ,  $\Xi$  also knows  $A'$ . The axiom maintains that knowledge is closed under material implication.
- Axiom D, also referred to as the *axiom of consistency* requires  $\Xi$  to have consistency in his knowledge: If an agent knows  $A$ , he does not simultaneously know its negation.<sup>4</sup>
- Axiom T, also called the *axiom of truth* or *axiom of veridicality*, says that if  $A$  is known by  $\Xi$ , then  $A$  is true.
- Axiom 4 is also known as the *axiom of self-awareness*, *positive introspection* or *KK-thesis*. They all refer to the idea that an agent has knowledge of his knowledge of  $A$  if he has knowledge of  $A$ .
- Axiom 5 is also known as the *axiom of wisdom*. It is the stronger thesis that an agent has knowledge of his own ignorance: If  $\Xi$  does not know  $A$ , he knows that he doesn't know  $A$ . The axiom is sometimes referred to as the *axiom of negative introspection*.
- Axiom .2 reveals that if  $\Xi$  does not know that he does not know  $A$ , then  $\Xi$  knows that he does not know not  $A$ .
- Axiom .3 maintains that either  $\Xi$  knows that his knowledge of  $A$  implies his knowledge of  $A'$  or he knows that his knowledge of  $A'$  implies his knowledge of  $A$ .
- Axiom .4 amounts to the claim that any true proposition *per se* constitutes knowledge and is sometimes referred to as *axiom of true (strong) belief*.

These axioms in proper combinations make up epistemic modal systems of varying strength depending on the modal formulae valid in the respective systems given the algebraic properties assumed for the accessibility relation.

The weakest system of epistemic interest is usually considered to be system **T**. The system includes **T** and **K** as valid axioms where **K** is valid in all Kripke-models. Additional modal strength may be obtained by extending **T** with other axioms drawn from the above pool altering the frame semantics to validate the additional axioms. By way of example, while  $K_{\exists}A \rightarrow A$  is valid in **T**,  $K_{\exists}A \rightarrow A$ ,  $K_{\exists}A \rightarrow K_{\exists}K_{\exists}A$  and  $\neg K_{\exists}A \rightarrow K_{\exists}\neg K_{\exists}A$  are all valid in **S5** but not in **T**. System **T** has a reflexive accessibility relation, **S5** has an equivalence relation of accessibility. The arrows in table 2 symbolize that the system to which the arrow is pointing is included in the system from which the arrow originates and hence reflect relative strength. Then **S5** is the strongest and **S4** the weakest of the ones listed.

<b>Epistemic Systems</b>		
KT4	=	<b>S4</b>
KT4 + .2	=	<b>S4.2</b> ↑
KT4 + .3	=	<b>S4.3</b> ↑
KT4 + .4	=	<b>S4.4</b> ↑
KT5	=	<b>S5</b> ↑

TABLE 2. Relative strength of epistemic systems between **S4** and **S5**

One of the important tasks of epistemic logic have been, and still is, to map the possible complete systems of such logics hopefully allowing for a picking of the most ‘appropriate’ ones even though this appropriateness may be highly context-dependent as Halpern has noted (10). These ‘appropriate’ logics often range from **S4** over the intermediate systems **S4.2-S4.4** to **S5**. By way of example, Hintikka settled for **S4** (16), Kutschera argued for **S4.4** (36), van der Hoek has proposed to strengthen knowledge according to system **S4.3** (34). In their contribution to this collection van Ditmarsch, van der Hoek and Kooi together with Fagin, Halpern, Moses and Vardi (6) assume knowledge to be **S5** valid.

In his contribution to this volume ‘Knowledge, Belief, and Subjective Probability: Outlines of a Unified System of Epistemic/Doxastic Logic’, Wolfgang Lenzen studies a range of axioms with respect to knowledge and two other attitudes expressing respectively conviction and belief, their formal properties and relations, philosophical implications and eventually concludes that:



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