Preface to the Second Edition

A teacher can never truly teach unless he is still learning himself. A lamp can never light another lamp unless it continues to burn its own flame. The teacher who has come to the end of this subject, who has no living traffic with his knowledge but merely repeats his lessons to his students, can only load their minds; he can not quicken them.

Rabindranath Tagore
Nobel Prize Winner for Literature (1913)

The first edition of this book was first published in 2002 under the sole authorship of Dr. Lokenath Debnath. It was well received and used as a senior undergraduate and first year graduate level text and reference in the USA and abroad for the last 12 years. We received various comments and suggestions from many students, faculty, and researchers around the world. These comments and criticisms have been very helpful, beneficial, and encouraging. The second edition is the result of these suggestions and comments.

The selection, arrangement, and presentation of the material in this edition have carefully been made based on our past and present teaching, research, and professional experience. In particular, this book has evolved from regularly teaching courses in wavelet transforms, signal analysis, differential equations, applied mathematics, and advanced engineering mathematics over many years to students of mathematics and engineering in the USA and abroad. It is essentially designed to cover advanced mathematical methods for science and engineering students with heavy emphasis to many different and varied applications. It differs from many textbooks with similar titles due to major emphasis placed on numerous topics and systematic development of the underlying theory before making applications and inclusion of many new and modern topics such as multiresolution analysis and construction of wavelets, extensions of multiresolution analysis, Gabor transforms and time–frequency signal analysis, the Wigner–Ville distribution, and time–frequency signal analysis. An attempt has also been made to provide a modern approach to fractals, turbulence, and Newland’s harmonic wavelets. Some of these new topics included in this second edition are not found in other texts and research reference books.
This extensively revised second edition preserves the basic content, style, and format of the first edition published in 2002. As with the previous edition, this book has been revised primarily as a comprehensive text for senior undergraduates and beginning graduate students and a research reference for professionals in mathematics, science, engineering, and other applied sciences. With basic prerequisites of calculus and ordinary differential equations, the main goal of the edition is to develop required analytical knowledge and skills on the part of the reader, rather than focus on the importance of more abstract formulation with full mathematical rigor. Indeed, our major emphasis is to provide an accessible working knowledge of the analytical and computational methods with proofs required in pure and applied mathematics, physics, and engineering.

Mathematics, science, and engineering students need to gain a sound knowledge of mathematical and computational skills. This book provides these by the systematic development of underlying theory with varied applications and by provision of carefully selected fully worked-out examples combined with their extensions and refinements through additions of a large set of a wide variety of exercises at the end of each chapter. Numerous standard and challenging topics, applications, worked-out examples, and exercises are included in this edition so that they stimulate research interest among senior undergraduates and graduate students. Another special feature of this book is to include sufficient modern topics which are vital prerequisites for subsequent advanced courses and research in mathematical, physical, and engineering sciences.

Readers familiar with the previous edition will notice many minor changes and numerous major ones in this edition. In general, changes have been made to modernize the contents and to improve the expositions and clarity of the previous edition to include additional materials, proofs, and comments as well as many examples of applications and exercises, and in some cases to entirely rewrite many sections. There is plenty of material in the book for long course, seminars, or workshops. Some of the materials need not be covered in a course work, seminars, or workshops and can be left for the readers to study on their own. This edition contains a collection of fully worked-out examples and challenging exercises with detailed answers and hints to many selected exercises. We have also updated the bibliography and corrected typographical errors. Major changes and additions include the following:

1. In Chap. 1, a brief historical introduction has been completely revised and expanded to include many new topics including the fractional Fourier transform, the fractional wavelet transform, the discrete wavelet transform, and the complex wavelet transform as well as the construction of wavelet bases in various spaces other than \( \mathbb{R} \) and several new extensions of the original multiresolution analysis. In addition, the last two decades have seen tremendous activity in the development of new mathematical and computational tools based on multiscale ideas such as steerable wavelets, wedgelets, beamlets, bandlets, ridgelets, curvelets, contourlets, surfacelets, shearlets, and platelets. This new historical discussion of the subject has been included to help the reader see the
directions in which the subject has developed and the new major contributions to its recent developments.

2. In order to make the book self-contained, Chap. 2 on Hilbert spaces and orthonormal systems with applications, and Chap. 3 on the theory of Fourier transforms and their diverse applications have been presented in great detail. Two new topics dealing with discrete Fourier transforms (DFT) and fast Fourier transforms (FFT) have been added to Chap. 3.

3. The Gabor transform and the Wigner–Ville distribution with time–frequency signal analysis are the major topics of Chaps. 4 and 5. Included are the Zak transform and its basic properties including the Balian–Low theorem and applications for studying the orthogonality and completeness of Gabor frames in the critical case. The relationship between the Wigner–Ville distribution and ambiguity functions is investigated with radar signal analysis. Recent generalizations of the Wigner–Ville distributions are briefly described.

4. Wavelet transforms and their basic properties are discussed in Chap. 6 in some detail. The discrete wavelet transforms and orthogonal wavelets are included in Chap. 6. Chapter 9 deals with Newland’s harmonic wavelets and their basic properties. Special attention is given to properties of harmonic scaling functions and Parseval’s formula for harmonic wavelets.

5. In Chap. 7, multiresolution analysis with examples and construction of wavelets are described in some detail. This chapter includes basic properties of scaling functions, bases of orthonormal wavelets, and construction of orthonormal wavelets. Special attention is given to Daubechies’ wavelet and algorithms, discrete wavelet transforms, and Mallat’s pyramid algorithms. In order to modernize the content of the book, two new major extensions of the original multiresolution analysis consisting of $p$-multiresolution analysis on the positive half-line and nonuniform multiresolution have been included in this new Chap. 8.

6. The final chapter deals with a brief discussion of the classical Fourier transform treatment of turbulence based on the Navier–Stokes equations and the equation of continuity. This is followed by a new treatment of certain aspects of turbulence based on fractals, multifractals, and singularities in turbulence. Included in this chapter is the modern approach to turbulence using the wavelet transform analyses. Special attention, in some detail, is given to Farge’s and Meneveau’s wavelet transform analyses of turbulent flows with the adaptive wavelet method of computation and analysis of turbulence.

7. The book offers a detailed and clear explanation of every concept and method that is introduced, accompanied by carefully selected worked-out examples with special emphasis being given to those topics in which students experience difficulty.

8. A wide variety of modern topics has been selected from areas of wavelet analysis, signal analysis, ordinary and partial differential equations, turbulence, multiresolution analysis and construction of wavelets, and Newland’s harmonic wavelets.
9. The book is organized with sufficient flexibility in teaching courses or directing seminars and workshops to enable instructors to select topics and chapters appropriate to courses of differing lengths, emphases, and levels of difficulty.

10. A wide spectrum of exercises has been carefully chosen and included at the end of each chapter, so the reader may further develop both analytical and computational skills in the theory and applications of wavelets, wavelet transform analysis, and multiresolution analysis and may gain a deeper insight into those subject.

11. The bibliography has been completely revised and updated. Many new research papers and standard books have been added to the bibliography to stimulate new interest in future advanced study and research. The index has also been completely revised in order to include a wide variety of topics.

12. The book provides information that puts the reader at the forefront of advanced study and current research.

With many improvements and numerous challenging topics, applications, and exercises, we hope this edition will continue to be a useful textbook for students as well as a research reference for professionals in mathematics, science, and engineering.

It is our pleasure to express our grateful thanks to many friends, colleagues, and students around the world who offered their suggestions and help at various stages of the preparation of the book. In spite of the best efforts of everyone involved, some typographical errors doubtless remain. Finally, we wish to express our thanks to Mrs. Danielle Walker, Associate Editor, and the staff of Birkhäuser-Springer for their help and cooperation.

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