Contents

Chapter 1
Hyperbolic Structures 1
1.1 The Hyperbolic Plane 1
1.2 Hyperbolic Structures 5
1.3 Pasting 8
1.4 The Universal Covering 15
1.5 Perpendiculars 17
1.6 Closed Geodesics 21
1.7 The Fenchel-Nielsen Parameters 27

Chapter 2
Trigonometry 31
2.1 The Hyperboloid Model 31
2.2 Triangles 33
2.3 Trirectangles and Pentagons 37
2.4 Hexagons 40
2.5 Variable Curvature 43
2.6 Appendix: The Hyperboloid Model Revisited 49
   The Quaternion Model 49
   A Trace Relation 55
   The General Sine and Cosine Formula 57

Chapter 3
Y-Pieces and Twist Parameters 63
3.1 Y-Pieces 63
3.2 Marked Y-Pieces 67
3.3 Twist Parameters 69
3.4 Signature (1, 1) 76
3.5 Cubic Graphs 78
3.6 The Compact Riemann Surfaces 81
3.7 Appendix: The Length Spectrum Is of Unbounded Multiplicity
   Geometric Approach
   Algebraic Approach

Chapter 4
The Collar Theorem
4.1 Collars
4.2 Non-Simple Closed Geodesics
4.3 Variable Curvature
4.4 Cusps
4.5 Triangulations of Controlled Size

Chapter 5
Bers’ Constant and the Hairy Torus
5.1 Bers’ Theorem
5.2 Partitions
5.3 The Hairy Torus
5.4 Bers’ Constant Without Curvature Bounds

Chapter 6
The Teichmüller Space
6.1 Marked Riemann Surfaces
6.2 Models of Teichmüller Space
6.3 The Real Analytic Structure of $\mathcal{F}_g$
6.4 Distances in $\mathcal{F}_g$
6.5 The Teichmüller Modular Group
6.6 A Rough Fundamental Domain
6.7 The Coordinates of Zieschang-Vogt-Coldewey
6.8 Fuchsian Groups and Bers’ Coordinates

Chapter 7
The Spectrum of the Laplacian
7.1 Introduction
7.2 The Spectrum and the Heat Equation
7.3 The Abel Transform
7.4 The Heat Kernel of the Hyperbolic Plane
7.5 The Heat Kernel of $\Gamma \backslash H$

Chapter 8
Small Eigenvalues
8.1 The Interval $[0, \frac{1}{4}]$
8.2 The Minimax Principles
8.3 Cheeger’s Inequality
8.4 Eigenvalue Estimates
Chapter 9
Closed Geodesics and Huber's Theorem 224

9.1 The Origin of the Length Spectrum 225
9.2 Summation over the Lengths 227
9.3 Summation over the Eigenvalues 235
9.4 The Prime Number Theorem 241
9.5 Selberg's Trace Formula 252
9.6 The Prime Number Theorem with Error Terms 256
9.7 Lattice Points 261

Chapter 10
Wolpert's Theorem 268

10.1 Introduction 268
10.2 Curve Systems 270
10.3 Finitely Many Lengths Determine the Length Spectrum 273
10.4 Generic Surfaces Are Determined by Their Spectrum 275
10.5 Decoding the Moduli 278

Chapter 11
Sunada's Theorem 283

11.1 Some History 283
11.2 Examples of Almost Conjugate Groups 285
11.3 Proof of Sunada's Theorem 291
11.4 Cayley Graphs 296
11.5 Transplantation of Eigenfunctions 304
11.6 Transplantation of Closed Geodesics 307

Chapter 12
Examples of Isospectral Riemann Surfaces 311

12.1 Cayley Graphs and Hyperbolic Polygons 311
12.2 Smoothness 313
12.3 Examples over \( \mathbb{Z}_g \times \mathbb{Z}_g \) 318
12.4 Examples over \( \text{SL}(3, 2) \) 321
12.5 Genus 6 325
12.6 Large Families 332
12.7 Criteria For Non-Isometry 333

Chapter 13
The Size of Isospectral Families 340

13.1 Finiteness 340
13.2 Parameter Geodesics of Length \( > \exp(-4g) \) 344
13.3 Measuring the Twist Parameters 347
13.4 Parameter Geodesics of Length \( \leq \exp(-4g) \) 355
Chapter 14
Perturbations of the Laplacian in Hilbert Space 362
14.1 The Hilbert Spaces $H_0$ and $H_1$ 362
14.2 The Friedrichs Extension of the Laplacian 366
14.3 A Representation Theorem 370
14.4 Resolvents and Projectors 373
14.5 Holomorphic Families 380
14.6 A Model of Teichmüller Space 382
14.7 Reduction to Finite Dimension 388
14.8 Holomorphic Families of Laplacians 397
14.9 Analytic Properties of the Eigenvalues 399
14.10 Finite Parts of the Spectrum 406

Appendix
Curves and Isotopies 409
The Theorems of Baer-Epstein-Zieschang 409
An Application to the 3-Holed Sphere 424
Length-Decreasing Homotopies 428

Bibliography 433
Index 448
Glossary 454
Geometry and Spectra of Compact Riemann Surfaces
Buser, P.
2010, XIV, 456 p. 145 illus., Softcover
ISBN: 978-0-8176-4991-3
A product of Birkhäuser Basel