Preface to Second Edition

For this second edition almost every chapter has been revised and updated in some way. The principal changes from the first edition are the following. A second chapter on the geometry of complex contact manifolds, Chapter 13, has been added. Chapter 7 on the curvature of contact metric manifolds has been reorganized and updated extensively. A section on the projectivized tangent bundle has been added to Chapter 9. Additional examples and commentary on further results have been added throughout.

There has been considerable work in recent years on a number of areas related to the subject, and it has been impossible to treat all of this work. The author believes, however, that the text offers a good introduction to and necessary background for the study of these topics.

The author expresses his appreciation to C. Abbas, T. Draghici, B. Foreman and B. Korkmaz for reading parts of the manuscript and offering valuable suggestions. The author also expresses his appreciation to Birkhäuser for suggesting that he write a second edition and especially to Ann Kostant, Jessica Belanger and Tom Grasso for their kind assistance in producing this book.

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David E. Blair
Preface to the First Edition

The author’s lectures “Contact Manifolds in Riemannian Geometry”, volume 509 (1976), in the Springer-Verlag series Lecture Notes in Mathematics have been out of print for some time and it seems appropriate that an expanded version of this material should become available. The present text deals with the Riemannian geometry of both symplectic and contact manifolds, although the book is more contact than symplectic. This work is based on the recent research of the author, his students, colleagues, and other scholars, the author’s graduate courses at Michigan State University and the earlier lecture notes.

Chapter 1 presents the general theory of symplectic manifolds. Principal circle bundles are then discussed in Chapter 2 as a prelude to the Boothby–Wang fibration of a compact regular contact manifold in Chapter 3, which deals with the general theory of contact manifolds. Chapter 4 focuses on Riemannian metrics associated to symplectic and contact structures. Chapter 5 is devoted to integral submanifolds of the contact subbundle. In Chapter 6 we discuss the normality of almost contact structures, Sasakian manifolds, K-contact manifolds, the relation of contact metric structures and CR-structures, and cosymplectic structures. Chapter 7 deals with the important study of the curvature of a contact metric manifold. In Chapter 8 we give a selection of results on submanifolds of
Kähler and Sasakian manifolds, including an illustration of the technique of A. Ros in a theorem of F. Urbano on compact minimal Lagrangian submanifolds in $\mathbb{CP}^n$. Chapter 9 discusses the symplectic structure of tangent bundles, contact structure of tangent sphere bundles, general vector bundles and normal bundles of Lagrangian and integral submanifolds giving rise to new examples of symplectic and contact manifolds. In Chapter 10 we study a number of curvature functionals on spaces of associated metrics and their critical point conditions; we show also that in the symplectic case, the “total scalar curvature” is a symplectic invariant and in the contact case is a natural functional whose critical points are the metrics for which the characteristic vector field generates isometries. In the presence of a certain amount of negative curvature, special directions appear in the contact subbundle; we discuss these and their relations to Anosov and conformally Anosov flows in Chapter 11. Chapter 12 deals with the subject of complex contact manifolds. We conclude with a brief treatment of 3-Sasakian manifolds in Chapter 13.

The text attempts to strike a balance between giving detailed proofs of basic properties, which will be instructive to the reader, and stating many results whose proofs would take us too far afield. It has been impossible, however, to be encyclopedic and include everything, so that unfortunately some important topics have been omitted or covered only briefly. An extensive bibliography is given.

It is the author’s hope that the reader will find this both a good introduction to the Riemannian geometry of contact and symplectic manifolds and a useful reference to recent research in the area.

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David E. Blair
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