## Contents

Preface ix

1 Nonlinear Elasticity 1
   1.1 Preliminary Considerations 1
   1.2 The Equilibrium Problem 2
   1.3 Remarks About Equilibrium Boundary Problems 4
   1.4 Variational Formulation of Equilibrium 7
   1.5 Isotropic Elastic Materials 11
   1.6 Homogeneous Deformations 12
   1.7 Homothetic Deformation 13
   1.8 Simple Extension of a Rectangular Block 16
   1.9 Simple Shear of a Rectangular Block 18
   1.10 Universal Static Solutions 21
   1.11 Constitutive Equations in Nonlinear Elasticity 24
   1.12 Treolar’s Experiments 25
   1.13 Rivlin and Saunders’ Experiment 26
   1.14 Nondimensional Analysis of Equilibrium 28
   1.15 Signorini’s Perturbation Method for Mixed Problems 29
   1.16 Signorini’s Method for Traction Problems 31
   1.17 Loads with an Equilibrium Axis 34
   1.18 Second-Order Hyperelasticity 36
   1.19 A Simple Application of Signorini’s Method 38
   1.20 Van Buren’s Theorem 40
   1.21 An Extension of Signorini’s Method to Live Loads 45
   1.22 Second-Order Singular Surfaces 47
   1.23 Singular Waves in Nonlinear Elasticity 51
   1.24 Principal Waves in Isotropic Compressible Elastic Materials 53
   1.25 A Perturbation Method for Waves in Compressible Media 56
   1.26 A Perturbation Method for Analyzing Ordinary Waves in Incompressible Media 60
2 Micropolar Elasticity 67
  2.1 Preliminary Considerations ................. 67
  2.2 Kinematics of a Micropolar Continuum ........ 68
  2.3 Mechanical Balance Equations ................ 73
  2.4 Energy and Entropy .......................... 76
  2.5 Elastic Micropolar Systems .................. 78
  2.6 The Objectivity Principle ................... 81
  2.7 Some Remarks on Boundary Value Problems ....... 86
  2.8 Asymmetric Elasticity ....................... 87

3 Continuous System with a Nonmaterial Interface 91
  3.1 Introduction ................................ 91
  3.2 Velocity of a Moving Surface ................. 92
  3.3 Velocity of a Moving Curve ................... 94
  3.4 Thomas’ Derivative and Other Formulae .......... 95
  3.5 Differentiation Formulae ...................... 96
  3.6 Balance Laws ................................ 101
  3.7 Entropy Inequality and Gibbs Potential ...... 106
  3.8 Other Balance Equations ...................... 109
  3.9 Integral Form of Maxwell’s Equations .......... 111

4 Phase Equilibrium 113
  4.1 Boundary Value Problems in Phase Equilibrium ... 113
  4.2 Some Phenomenological Results of Changes in State ... 114
  4.3 Equilibrium of Fluid Phases with a Planar Interface ... 117
  4.4 Equilibrium of Fluid Phases with a Spherical Interface ... 119
  4.5 Variational Formulation of Phase Equilibrium ...... 122
  4.6 Phase Equilibrium in Crystals ................ 125
  4.7 Wulff’s Construction ........................ 130

5 Stationary and Time-Dependent Phase Changes 133
  5.1 The Problem of Continuous Casting .......... 133
  5.2 On the Evolution of the Solid–Liquid Phase Change ... 138
  5.3 On the Evolution of the Liquid–Vapor Phase Change ... 142
  5.4 The Case of a Perfect Gas .................. 146

6 An Introduction to Mixture Theory 149
  6.1 Balance Laws ............................... 150
  6.2 Classical Mixtures .......................... 155
  6.3 Nonclassical Mixtures ........................ 159
  6.4 Balance Equations of Binary Fluid Mixtures .... 161
  6.5 Constitutive Equations ...................... 163
  6.6 Phase Equilibrium and Gibbs’ Principle .......... 167
  6.7 Evaporation of a Fluid into a Gas ............ 168
7 Electromagnetism in Matter
7.1 Integral Balance Laws ..................................... 171
7.2 Electromagnetic Fields in Rigid Bodies at Rest ............ 174
7.3 Constitutive Equations for Isotropic Rigid Bodies .......... 178
7.4 Approximate Constitutive Equations for Isotropic Bodies . 180
7.5 Maxwell’s Equations and the Principle of Relativity ....... 181
7.6 Quasi-electrostatic and Quasi-magnetostatic
    Approximations ........................................... 185
7.7 Balance Equations for Quasi-electrostatics ................. 189
7.8 Isotropic and Anisotropic Constitutive Equations .......... 192
7.9 Polarization Fields and the Equations of Quasi-electrostatics 194
7.10 More General Constitutive Equations ...................... 197
7.11 Lagrangian Formulation of Quasi-electrostatics ............ 198
7.12 Variational Formulation for Equilibrium in Quasi-
    electrostatics ........................................... 201

8 Introduction to Magnetofluid Dynamics
8.1 An Evolution Equation for the Magnetic Field ............... 205
8.2 Balance Equations in Magnetofluid Dynamics ............... 207
8.3 Equivalent Form of the Balance Equations .................. 208
8.4 Constitutive Equations .................................... 211
8.5 Ordinary Waves in Magnetofluid Dynamics .................. 212
8.6 Alfven’s Theorems ......................................... 216
8.7 Laminar Motion Between Two Parallel Plates ................ 217
8.8 Law of Isorotation ........................................ 222

9 Continua with an Interface and Micromagnetism
9.1 Ferromagnetism and Micromagnetism ....................... 225
9.2 A Ferromagnetic Crystal as a Continuum with an Interface 227
9.3 Variations in Surfaces of Discontinuity .................... 228
9.4 Variational Formulation of Weiss Domains ................ 229
9.5 Weiss Domain Structure .................................... 231
9.6 Weiss Domains in the Absence of a Magnetic Field ........ 234
9.7 Weiss Domains in Uniaxial Crystals ........................ 236
9.8 A Variational Principle for Elastic Ferromagnetic Crystals 239
9.9 Weiss Domains in Elastic Uniaxial Crystals ................ 241
9.10 A Possible Weiss Domain Distribution in Elastic Uniaxial
    Crystals .................................................. 243
9.11 A More General Variational Principle ...................... 244
9.12 Weiss Domain Branching .................................... 250
9.13 Weiss Domains in an Applied Magnetic Field ............... 252
## Contents

10 Relativistic Continuous Systems 257
10.1 Lorentz Transformations .................................. 257
10.2 The Principle of Relativity .................................. 261
10.3 Minkowski Spacetime ........................................ 264
10.4 Physical Meaning of Minkowski Spacetime ................. 268
10.5 Four-Dimensional Equation of Motion .................. 270
10.6 Integral Balance Laws ...................................... 272
10.7 The Momentum–Energy Tensor ............................ 274
10.8 Fermi and Fermi–Walker Transport ....................... 277
10.9 The Space Projector .......................................... 281
10.10 Intrinsic Deformation Gradient ........................... 283
10.11 Relativistic Dissipation Inequality ....................... 286
10.12 Thermoelastic Materials in Relativity ................. 289
10.13 About the Physical Meanings of Relative Quantities ... 293
10.14 Maxwell’s Equation in Matter ............................. 295
10.15 Minkowski’s Description .................................. 297
10.16 Amperé’s Model ............................................. 298

A Brief Introduction to Weak Solutions 301
A.1 Weak Derivative and Sobolev Spaces ...................... 301
A.2 A Weak Solution of a PDE .................................. 305
A.3 The Lax–Milgram Theorem .................................. 307

B Elements of Surface Geometry 309
B.1 Regular Surfaces ............................................. 309
B.2 The Second Fundamental Form .............................. 311
B.3 Surface Gradient and the Gauss Theorem .................. 316

C First-Order PDE 319
C.1 Monge’s Cone ............................................... 319
C.2 Characteristic Strips ......................................... 321
C.3 Cauchy’s Problem ........................................... 324

D The Tensor Character of Some Physical Quantities 327

References 331

Index 345