Preface

In the companion book (Continuum Mechanics Using Mathematica®) to this volume, we explained the foundations of continuum mechanics and described some basic applications of fluid dynamics and linear elasticity. However, deciding on the approach and content of this book, Continuum Mechanics: Advanced Topics and Research Trends, proved to be a more difficult task. After a long period of reflection, we made the decision to direct our efforts into drafting a book that demonstrates the flexibility and great potential of continuum physics to describe the wide range of macroscopic phenomena that we can observe. It is the opinion of the authors that this is the most stimulating way to learn continuum mechanics. However, it is also quite evident that this aim cannot be fully realized in a single book. Consequently, in this book we chose to present only the basics of interesting continuum mechanics models, along with some important applications of them.

We assume that the reader is familiar with all of the basic principles of continuum mechanics: the general balance laws, constitutive equations, isotropy groups for materials, the laws of thermodynamics, ordinary waves, etc. All of these concepts can be found in Continuum Mechanics Using Mathematica and many other books.

We believe that this book gives the reader a sufficiently wide view of the “boundless forest” of continuum mechanics, before focusing his or her attention on the beauty and complex structure of single trees within it (indeed, we could say that Continuum Mechanics Using Mathematica provides only the fertile humus on which the trees of this forest take root!).

The topics that we have selected for this book in order to show the power of continuum mechanics to characterize the experimental behavior of real bodies, and the order in which these topics are discussed here, are described below.

In Chap. 1, we discuss some interesting aspects of nonlinear elasticity. We start with the equilibrium equations and their variational formulation and discuss some peculiarities of the boundary value problems of
nonlinear elasticity. We then analyze the homogeneous equilibrium solutions of isotropic materials together with the universal equilibrium solutions of Ericksen for compressible elastic materials. Moreover, some experimental results for constitutive equations in nonlinear elasticity are briefly explored. The existence and uniqueness theorems of Van Buren and Stoppelli, as well as Signorini’s method, are presented with some recent extensions to live loads. Finally, the chapter concludes with a survey of the propagation of acceleration waves in an elastic body, and a new perturbation method for the analysis of these waves is presented.

In Chap. 2, we discuss the theory of continua with directors, which was proposed at the beginning of the twentieth century by the Cosserat brothers and was subsequently developed by many other authors. In this model, a continuous system $S$ is no longer considered a collection of simple points defined by their coordinates in a frame of reference; instead, $S$ is regarded as a set of complex particles that also possess a certain number of vectors that move independently of the particles with which they are associated. Such a model provides a better description of aggregates of microcrystals, polarized dielectrics, ferromagnetic substances, and one-dimensional and two-dimensional bodies. It can also be applied whenever the system contains a length that: (i) is less than the limit considered in continuum mechanics; (ii) characterizes the dimensions of microscopic regions that influence the macroscopic behavior of the body through their internal evolutions.

In Chap. 3, we consider a simplified model of a continuum with a nonmaterial moving surface across which the bulk fields can exhibit discontinuities. The general balance equations of this model are formulated together with the associated local field equations and jump conditions. In Chap. 4, this model is used to describe the phase equilibrium of two different phases. In particular, Maxwell’s rule and Clapeyron’s equation are derived.

The same model is applied in Chap. 5 to describe dynamical phase changes like melting and evaporation. The related difficult free-boundary problems are stated together with some numerical results.

Chapter 6 introduces the principles of mixture theory. This model, which allows us to describe the evolution of each constituent of a mixture as well as the whole mixture, is very useful in chemistry, biology, and mineralogy (alloys). This chapter contains a proof for the Gibbs rule, together with an analysis of phase equilibrium in a binary mixture.

Chapters 7 and 8 describe the interactions of electric and magnetic fields with matter using a continuum model with a nonmaterial interface. After a general discussion of the different properties resulting from a change of reference frame for the mechanical and electromagnetic equations, the approximations of quasi-electrostatics and quasi-magnetostatics are discussed. In particular, by adopting a continuum mechanics approach, we show that various physical models that have been proposed to explain the behavior of dielectrics and magnetic bodies are actually equivalent from a macroscopic
perspective. In other words, different microscopic models can lead to the same macroscopic behavior.

In Chap. 9, we present the macroscopic approach to micromagnetism together with the very difficult mathematical problems associated with this model. Among other things, it is shown that the model of a continuum with a nonmaterial interface can be used to determine the form of Weiss’ domains for some crystals and geometries.

Chapter 10 provides an introduction to continua in special relativity. After a brief analysis of the historical motivations of this theory, Minkowski’s geometrical model of spacetime is presented. The relativistic balance equations are then formulated in terms of the symmetric momentum–energy four-tensor. After an accurate description of Fermi transport, the intrinsic deformation gradient is introduced, in order to define elastic materials by extending the objectivity principle to special relativity. We then justify the different transformation formulae adopted in the literature for the total work, the total energy and the total heat of an homogeneous system through a wide-ranging discussion of the absolute and relative viewpoints. At the end of this chapter, the fundamental problem of the interaction between matter and electromagnetic fields is analyzed, together with the different models that have been adopted to describe it. Finally, we prove the equivalence of all of these proposals.

There are only a few notebooks written in Mathematica\textsuperscript{®} for this book (which can be downloaded from the publisher’s website at http://www.birkhauser.com/978-0-8176-4869-5), since the topics here discussed are more theoretical in nature than those treated in Continuum Mechanics Using Mathematica. However, many of the notebooks associated with that book can also be applied to the topics covered here.

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